Continuum Relational Physics in Group Field Theories and Applications to Cosmology

Luca Marchetti

Quantum Gravity Group Seminars
Perimeter Institute, Waterloo
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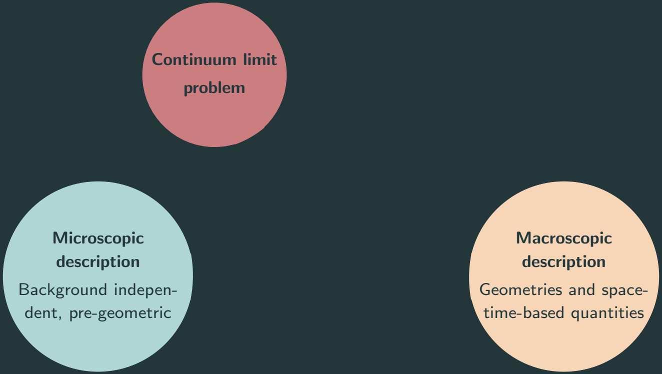
Department of Mathematics and Statistics
UNB Fredericton

Microscopic description

Background indepen-
dent, pre-geometric

Macroscopic description

Geometries and space-
time-based quantities


Continuum limit problem
Microscopic description
Background independent, pre-geometric
Macroscopic description
Geometries and space-time-based quantities


Continuum limit problem
Microscopic description
Background independent, pre-geometric
Macroscopic description
Geometries and spacetime-based quantities
Localization problem


Coarse-graining
Microscopic description
Background independent, pre-geometric
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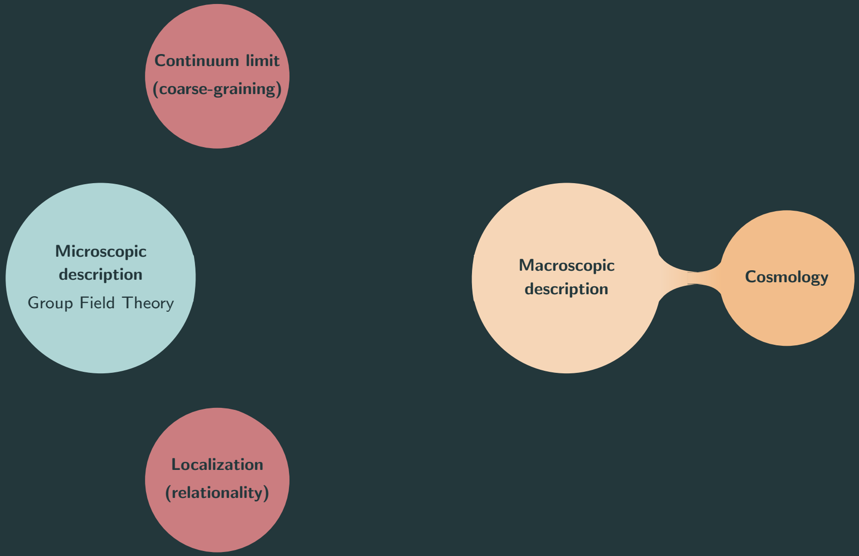
Relationality


Continuum limit
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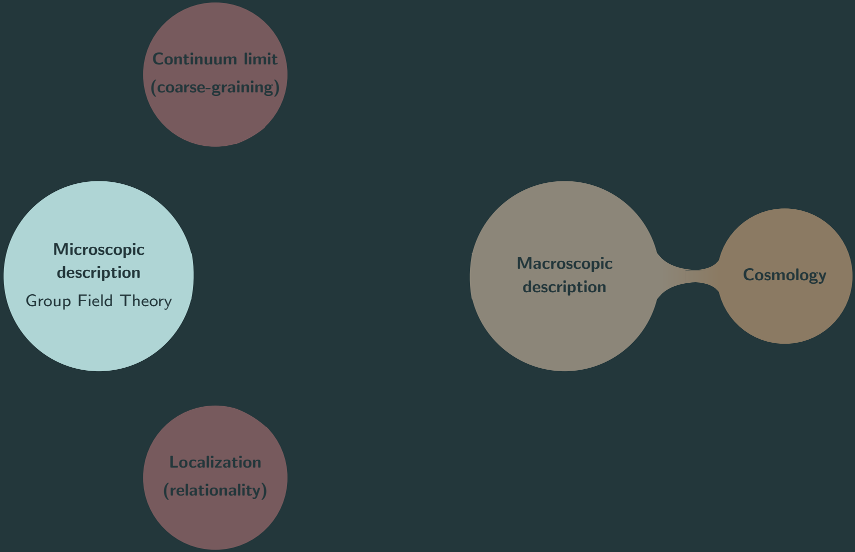
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Introduction to GFTs

Group Field Theory and simplicial gravity

Definition

Group Field Theories: theories of a field $\varphi : G^r \to \mathbb{C}$ defined on r copies of a group manifold G.

r is the dimension of the "spacetime to be" ($r = 4$) and G is the local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or, for some models, $G = \mathrm{SU}(2)$.

Oriti 1110.5606; Reisenberger, Rovelli 0002083; Freidel 0505016; Baratin, Oriti 1002.4723; Finocchiaro, Oriti 1812.03550.

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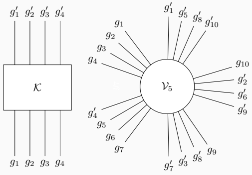
Action

$$S[\varphi, \bar{\varphi}] = \int \mathrm{d}g_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_\gamma \frac{\lambda_\gamma}{n_\gamma} \mathrm{Tr}_{\mathcal{V}_\gamma}[\varphi] + \text{c.c.}\,.$$

- Interaction terms are combinatorially non-local.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ:

$$\mathrm{Tr}_{\mathcal{V}_\gamma}[\varphi] = \int \prod_{i=1}^{n_\gamma} \mathrm{d}g_a \prod_{(a,i;b,j)} \mathcal{V}_\gamma(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_\gamma} \varphi(g_a^{(i)})\,.$$



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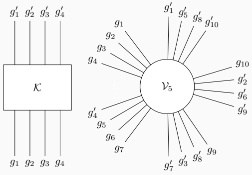

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Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_\Gamma(\{\lambda_\gamma\}) A_\Gamma$$

- Γ = stranded diagrams dual to r-dimensional cellular complexes of arbitrary topology.
- Amplitudes A_Γ = sums over group theoretic data associated to the cellular complex.

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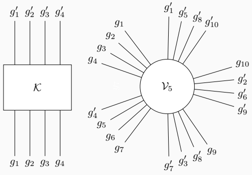

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Partition function

$$Z[\varphi,\bar\varphi] = \sum_\Gamma w_\Gamma(\{\lambda_\gamma\})A_\Gamma = \text{complete spin foam model.}$$

- Γ = stranded diagrams dual to r-dimensional cellular complexes of arbitrary topology.
- Amplitudes A_Γ = sums over group theoretic data associated to the cellular complex.
- $\mathcal{K}$ and $\mathcal{V}_\gamma$ chosen to match the desired spin foam model.

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Fundamental quanta

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^{4} \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Finocchiaro, Oriti 1812.03550; Baez, Barrett 9903060; Baratin, Oriti 1002.4723; Gielen, Oriti 1004.5371; Oriti 1310.7786.

Group Field Theory and Loop Quantum Gravity

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Lie algebra (metric)

$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$

Constraints

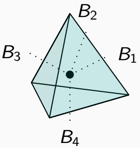
Geometricity constraints (appropriately encoded in $\mathcal{K}$ and $\mathcal{V}_\gamma$) allow for a $r-1$-simplicial interpretation of the fundamental quanta:

Closure

$\sum_a B_a = 0$
(faces of the tetrahedron close).

Simplicity

- $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
- $X \cdot B_a = 0$ (BC).



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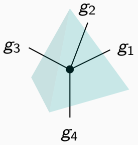

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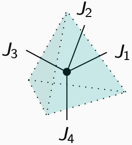

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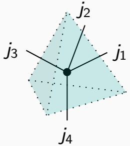

LQG

- Impose simplicity and reduce to $G = \mathrm{SU}(2)$.
- Impose closure (gauge invariance).

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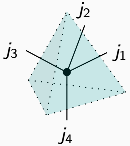

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$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv}\left[\bigotimes_{a=1}^{4} \mathcal{H}_{j_a}\right]$$
$$= \text{open spin-network vertex space}$$

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The Group Field Theory Fock space

Tetrahedron wavefunction

$\varphi(g_1, \dots, g_4)$
(subject to constraints)

Oriti 1310.7786; Oriti 1408.7112; Sahlman, Sherif 2302.03612.

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$\xrightarrow[\text{Theory}]{\text{Many-body}}$

GFT field operator

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GFT Fock space

$$\mathcal{F}_{\mathsf{GFT}} = \bigoplus_{V=0}^{\infty} \mathrm{sym}\left[\mathcal{H}^{(1)}_{\mathsf{tetra}} \otimes \mathcal{H}^{(2)}_{\mathsf{tetra}} \otimes \dots \mathcal{H}^{(V)}_{\mathsf{tetra}}\right]$$

- $\mathcal{F}_{\mathsf{GFT}}$ generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- $\mathcal{H}_\Gamma \subset \mathcal{F}_{\mathsf{GFT}}$, $\mathcal{H}_\Gamma$ space of states associated to connected simplicial complexes Γ.
- Generic states do not correspond to connected simplicial lattices nor classical simplicial geometries.
- Similar to $\mathcal{H}_{\mathsf{LQG}}$ but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

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Operators

Volume operator $\hat{V} = \int \mathrm{d}g_a^{(1)}\,\mathrm{d}g_a^{(2)}\, V(g_a^{(1)}, g_a^{(2)})\hat{\varphi}^\dagger(g_a^{(1)})\hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \iota} V_{j_a,\iota}\hat{\varphi}^\dagger_{j_a,m_a,\iota}\hat{\varphi}_{j_a,m_a,\iota}$.

- Generic second quantization prescription to build a $m+n$-body operator: sandwich matrix elements between spin-network states between m powers of $\hat{\varphi}^\dagger$ and n powers of $\hat{\varphi}$.

Oriti 1310.7786; Oriti 1408.7112; Sahlman, Sherif 2302.03612.

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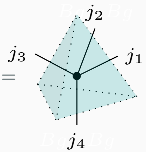
Kinematics

Quanta are $r - 1$-simplices decorated with quantum geometric data:

- Geometricity constraints imposed analogously as before.

$\mathcal{H}_{\text{tetra}} =$ 

Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

- Geometric data enter the action in a non-local and combinatorial fashion.

Group Field Theory and matter: scalar fields

Group Field Theories: theories of a field $\varphi : G^r \times \mathbb{R}^{d_l} \to \mathbb{C}$ defined on the product of G^r and $\mathbb{R}^{d_l}$.

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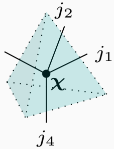
Kinematics

Quanta are $r-1$-simplices decorated with quantum geometric and scalar data:

- Geometricity constraints imposed analogously as before.
- Scalar field discretized on each d-simplex: each $d-1$-simplex composing it carries values $\boldsymbol{\chi} \in \mathbb{R}^{d_l}$.

$\mathcal{H}_{\text{tetra}} =$ 

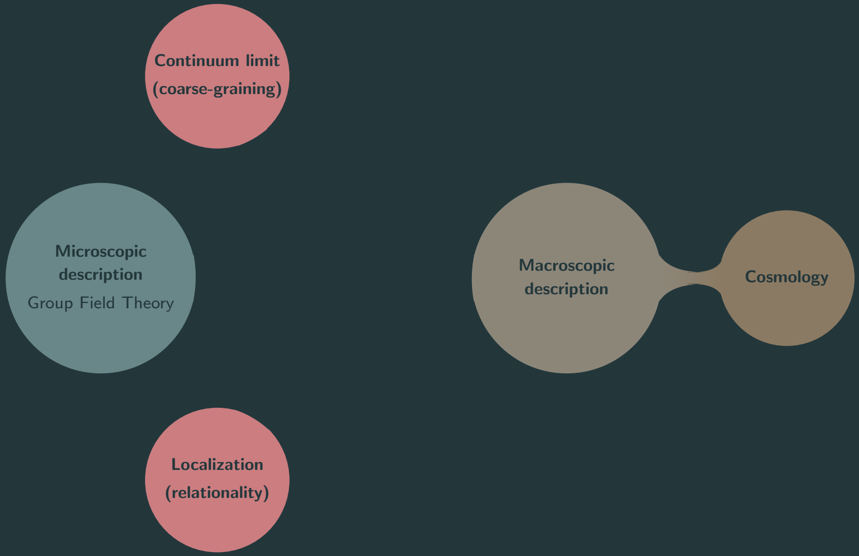
Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- Geometric data enter the action in a non-local and combinatorial fashion.
- Scalar field data are local in interactions.
- For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi^\alpha, \chi^{\alpha\prime}) = \mathcal{K}(g_a, g_b; (\chi^\alpha - \chi^{\alpha\prime})^2)$$

$$\mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)}, \boldsymbol{\chi}) = \mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)})$$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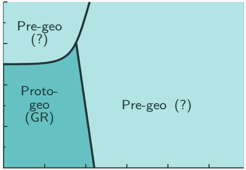
Continuum limit and localization

The continuum limit problem

Continuum physics and QG: the general perspective

Continuum limit in QG ⟷ Large number of dynamical microscopic dof.

Full partition function needed!



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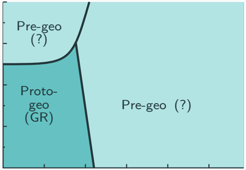

The (F)RG perspective

QFT on spacetime

- Energy scale defines the flow from IR and UV.

QG theory

- Only internal scales (localization problem).

LM, Oriti, Pithis, Thürigen 2211.12768-2209.04297-2110.15336; Oriti 2112.02585, Finocchiaro, Oriti 2004.07361; Reuter, Saueressig 2019; . . .

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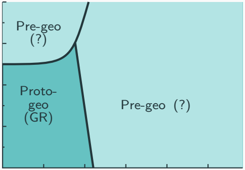

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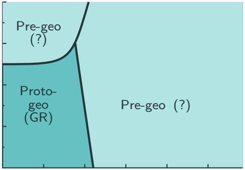

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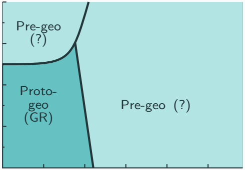

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Little control over QG theory space!

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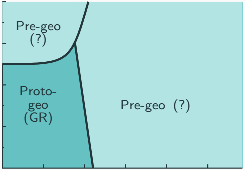

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Little control over QG theory space!

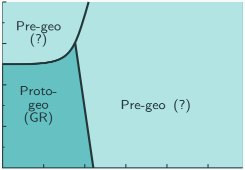
Approximate methods

LM, Oriti, Pithis, Thürigen 2211.12768-2209.04297-2110.15336; Oriti 2112.02585, Finocchiaro, Oriti 2004.07361; Reuter, Saueressig 2019; . . .

Continuum physics and QG: the general perspective

Continuum limit in QG ⟷ Large number of dynamical microscopic dof.

Full partition function needed!



The (F)RG perspective

QFT on spacetime

- Energy scale defines the flow from IR and UV.

QG theory

- Only internal scales (localization problem).

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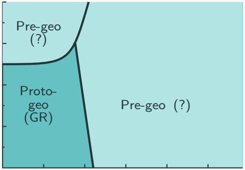

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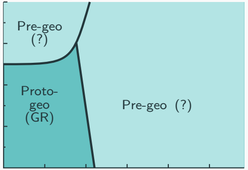

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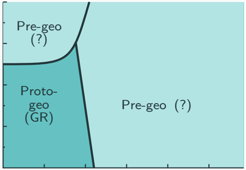

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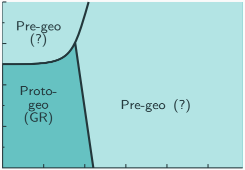

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Mean-field approximation in GFT

Local theory

Mean-field

- Mass $m^2 \equiv \mu$; interactions $\lambda\varphi^4$.
- Two different phases:

$$\frac{\delta S}{\delta \varphi} = 0 \;:\; \begin{cases} \varphi_0 = 0\,, & \mu > 0\,, \\ \varphi_0 \neq 0\,, & \mu < 0\,. \end{cases}$$

LM, Oriti, Pithis, Thürigen 2211.12768-2209.04297-2110.15336.

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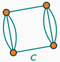

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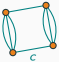

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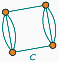

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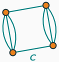

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- Matter (scalars): local $G_l = \mathbb{R}^{d_l}$.
- BC: $G = \mathrm{SL}(2,\mathbb{C})$+constraints.
- $0 \leq \eta < L$ & Wick rotation.

Analogous to toy GFT.

- $d = d_l + d_g(r - s_0)$.
- $d = d(\xi) \underset{\xi\to\infty}{\longrightarrow} \infty$!
- Flat limit as above.

Universal feature

Mean-field theory is always a good description of the phase transition!

LM, Oriti, Pithis, Thürigen 2211.12768-2209.04297-2110.15336.

The localization problem

Relational strategy and emergent quantum gravity theories

Background independence $\longleftrightarrow$ **Problem of localization** $\longleftrightarrow$ **Relational strategy**

Quite well understood from a classical perspective, less from a quantum perspective.

LM, Oriti 2008.02774; Rovelli Class. Quantum Grav. 8 297; Dittrich 0507106; Goeller, Höhn, Kirklin 2206.01193; ...

Relational strategy and emergent quantum gravity theories

Background independence ⟷ **Problem of localization** ⟷ **Relational strategy**

A genuinely new dimension of the problem arises for emergent QG theories.

Microscopic pre-geo

- Fundamental d.o.f. are weakly related to spacetime quantities;
- The latter expected to emerge from the former when a continuum limit is taken.

Macroscopic proto-geo

- Set of collective observables;
- Coarse grained states or probability distributions.

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Deeply intertwined with continuum limit problem!

Effective approaches!

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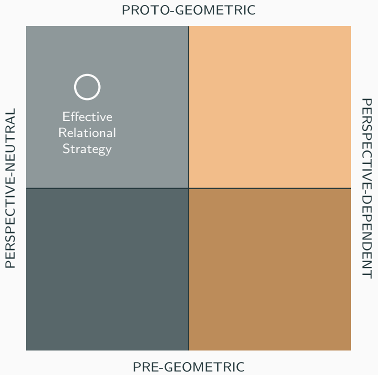
Emergent effective relational strategy



Basic principles

Emergence Relational strategy in terms of collective observables and states.

Effectiveness Averaged relational localization. Internal frame not too quantum.

LM, Oriti 2008.02774; Bojowald, Hoehn, Tsobanjan 1011.3040; Bojowald, Tsobanjan 0906.1772;

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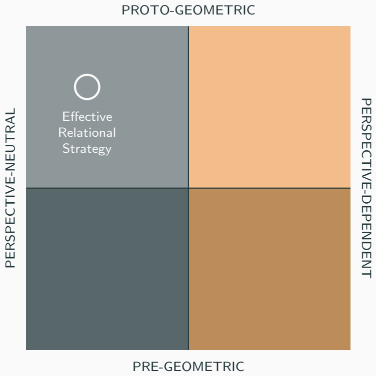

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Concrete example: scalar field clock

Emergence

- Identify (collective) states $|\Psi\rangle$ admitting a continuum proto-geometric interpretation.
- Identify a set of collective observables:

$$\hat{O}_a \xrightarrow{\langle\cdot\rangle_\Psi} \text{Geometric observables} \qquad \hat{\chi} \quad \hat{\Pi} \xrightarrow{\langle\cdot\rangle_\Psi} \text{Scalar field and its momentum} \qquad \hat{N} \xrightarrow{\langle\cdot\rangle_\Psi} \text{Number of quanta}$$

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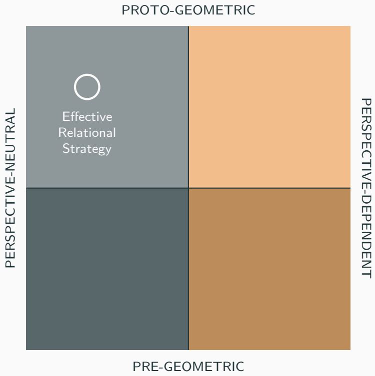

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Effectivness

- It exists a "Hamiltonian" $\hat{H}$ such that

$$i\frac{\mathrm{d}}{\mathrm{d}\,\langle\hat{\chi}\rangle_\Psi}\langle\hat{O}_a\rangle_\Psi = \langle[\hat{H},\hat{O}_a]\rangle_\Psi\ ,$$

  and whose moments coincide with those of $\hat{\Pi}$.
- Relative fluctuations of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$.

$$\Delta^2\chi \ll 1\,, \qquad \Delta^2\chi \sim \langle\hat{N}\rangle_\Psi^{-1}\ .$$

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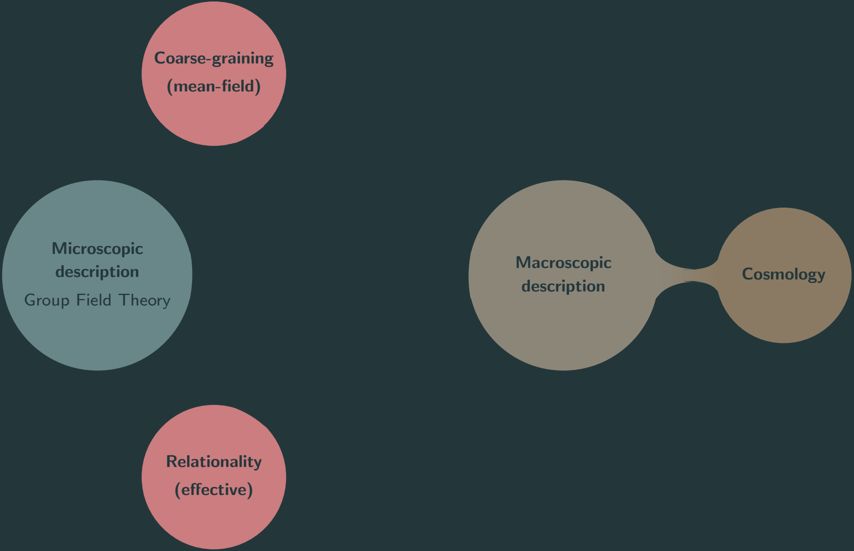
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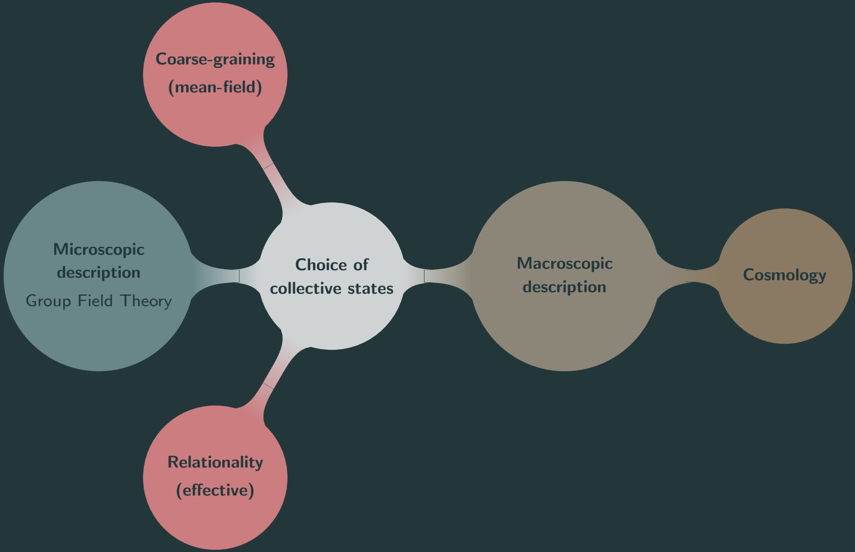
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Quantum gravity coherent states

GFT coherent states

- From the GFT perspective, continuum geometries are associated to large number of quanta.
- The simplest states that can accommodate infinite number of quanta are coherent states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp\left[\int \mathrm{d}^{d_l}\chi \int \mathrm{d}g_a\, \sigma(g_a, \chi^\alpha)\hat{\varphi}^\dagger(g_a, \chi^\alpha)\right]|0\rangle\,.$$

LM, Oriti 2008.02774-2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238.

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Mean-field approximation

- When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S_{\mathrm{GFT}}[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta\hat{\varphi}(g_I, x^\alpha)} \right\rangle_\sigma = \int \mathrm{d}h_a \int \mathrm{d}\chi\, \mathcal{K}(g_a, h_a, (x^\alpha - \chi^\alpha)^2)\sigma(h_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta\varphi^*(g_a, x^\alpha)}\bigg|_{\varphi=\sigma} = 0\,.$$

- Non-perturbative: equivalent to a mean-field (saddle-point) approximation of Z.

LM, Oriti 2008.02774-2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238.

Quantum gravity coherent states

Collective states

GFT coherent states

- From the GFT perspective, continuum geometries are associated to large number of quanta.
- The simplest states that can accommodate infinite number of quanta are coherent states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp\left[\int \mathrm{d}^{d_l}\chi \int \mathrm{d}g_a\, \sigma(g_a,\chi^\alpha)\hat{\varphi}^\dagger(g_a,\chi^\alpha)\right]|0\rangle\,.$$

Coarse-graining

Mean-field approximation

- When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S_{\mathrm{GFT}}[\hat{\varphi},\hat{\varphi}^\dagger]}{\delta\hat{\varphi}(g_I,x^\alpha)}\right\rangle_\sigma = \int \mathrm{d}h_a \int \mathrm{d}\chi\, \mathcal{K}(g_a,h_a,(x^\alpha-\chi^\alpha)^2)\sigma(h_a,\chi^\alpha) + \lambda \left.\frac{\delta V[\varphi,\varphi^*]}{\delta\varphi^*(g_a,x^\alpha)}\right|_{\varphi=\sigma} = 0\,.$$

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Localization

Relational peaking

- Relational localization implemented at an effective level on observable averages. E.g., χ^μ-frame:

$$\sigma_x = (\text{fixed peaking function } \eta_x) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})\,,$$

$$\mathcal{O}(x) \equiv \langle\hat{\mathcal{O}}\rangle_{\sigma_x} \simeq \mathcal{O}[\tilde{\sigma}]|_{\chi^\mu = x^\mu}$$

$$\langle\hat{\chi}^\mu\rangle_{\sigma_x} \simeq x^\mu$$

e.g.

$$\hat{N} = \int \mathrm{d}g_a\, \mathrm{d}^4\chi^\mu\, \hat{\varphi}^\dagger(g_a,\chi^\mu)\hat{\varphi}(g_a,\chi^\mu)$$

$$N(x) = \int \mathrm{d}g_a\, |\tilde{\sigma}(g_a,x^\mu)|^2$$

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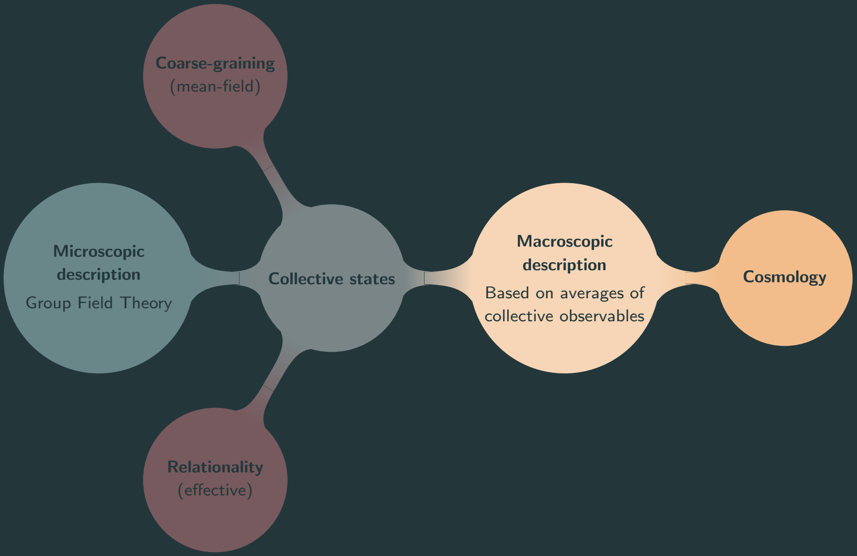
Coarse-graining
(mean-field)
Microscopic description
Group Field Theory
Collective states
Relationality
(effective)
Macroscopic description
Based on averages of collective observables
Cosmology

FLRW sector

Effective FLRW cosmological dynamics

Effective dynamics

Mean-field approximation

- Homogeneity: $\tilde{\sigma}$ depends only on MCMF clock χ^0.
- Isotropy: $\tilde{\sigma}_\upsilon \equiv \rho_\upsilon e^{i\theta_\upsilon}$ ($\upsilon_{\text{EPRL}} \in \mathbb{N}/2$, $\upsilon_{\text{BC}} \in \mathbb{R}$).
- Mesoscopic regime: negligible interactions.

$$0 = \tilde{\sigma}_\upsilon'' - 2i\tilde{\pi}_0\tilde{\sigma}_\upsilon' - E_\upsilon^2\tilde{\sigma},$$

$$V(x^0) = \sum\!\!\!\!\!\!\int_\upsilon V_\upsilon |\tilde{\sigma}_\upsilon|^2(x^0).$$

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$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2\sum_\upsilon V_\upsilon\rho_\upsilon \operatorname{sgn}(\rho'_\upsilon)\sqrt{\mathcal{E}_\upsilon - Q^2_\upsilon/\rho^2_\upsilon + \mu^2_\upsilon\rho^2_\upsilon}}{3\sum_\upsilon V_\upsilon\rho^2_\upsilon}\right)^2, \qquad \frac{V''}{V} = \frac{2\sum_\upsilon V_\upsilon\left[\mathcal{E}_\upsilon + 2\mu^2_\upsilon\rho^2_\upsilon\right]}{\sum_\upsilon V_\upsilon\rho^2_\upsilon}$$

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Classical limit

Large number of quanta (large volume and late times)

- ✓ Volume quantum fluctuations under control.
- If μ^2_υ is mildly dependent on υ (or one υ_o is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

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$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow$ flat FLRW

- ✓ Classical limit seems universal!
- ✓ $x^0 = \langle \hat{\chi}^0 \rangle_{\sigma_{x^0}}$, clock quantum fluct. $\simeq 0$.
- ✓ $\langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}} = \langle \hat{H}_\sigma \rangle_{\sigma_{x^0}}$ (higher moments $\simeq 0$).

Effective relational framework reliable!

LM, Oriti 2008.02774-2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Oriti, Pithis 2112.00091.

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Quantum bounce

Smaller number of quanta (smaller volume and early times)

- For a large range of initial conditions (at least one $Q_\upsilon \neq 0$ or one $\mathcal{E}_\upsilon < 0$)

Singularity res. into quantum bounce!

LM, Oriti 2008.02774-2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Oriti, Pithis 2112.00091.

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- For a large range of initial conditions (at least one $Q_\upsilon \neq 0$ or one $\mathcal{E}_\upsilon < 0$)
- Volume quantum fluctuations may be large!

Singularity res. into quantum bounce?

- x^0 may not coincide with $\langle\hat{\chi}^0\rangle_{\sigma_{x^0}}$ anymore!
- Clock quantum fluctuations may be large!
- $\langle\hat{\Pi}^0\rangle_{\sigma_{x^0}} \neq \langle\hat{H}_\sigma\rangle_{\sigma_{x^0}}$ (higher moments $\neq 0$).

Effective rel. framework may break down!

LM, Oriti 2008.02774-2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Oriti, Pithis 2112.00091.

A state-agnostic approach

Quantum system

Construction of the effective system

Step 1: definition of the quantum phase space

- Describe the system with $\langle \hat{A}_i \rangle$ and moments.
- Inherited Poisson structure: $\{\langle\cdot\rangle\,,\langle\cdot\rangle\}=(i\hbar)^{-1}\langle[\cdot,\cdot]\rangle$

Step 2: definition of the constraints

- $\langle \hat{C} \rangle = 0$ and $\langle (\widehat{\mathrm{pol}} - \langle \widehat{\mathrm{pol}} \rangle) \hat{C} \rangle = 0$ eff. constraints;

Step 3: truncation scheme (e.g. semiclassicality)

LM, Gielen, Oriti, Polaczek 2110.11176; Bojowald, Sandhoefer, Skirzewski, Tsobanjan 0804.3365; Bojowald Tsobanjan 0906.1772.

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Relational description

Step 1: **choose a clock** $\hat{T}$ ($[\hat{T}, \hat{P}]$ closes)

Step 2: **gauge fixing**

- 1st order: $\Delta(TA_i) = 0$, $A_i \in \mathcal{A} \setminus \{\hat{P}\}$.

Step 3: **relational rewriting**

- Write evolution of the remaining variables wrt. T (classical clock).

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Infinitely many algebra generators.

Infinitely many quantum constraints.

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- Interest in a coarse grained system characterized by a small number of macroscopic (1-body) observables.
- Expected to be the case for cosmology.

Coarse-graining truncation

- When the e.o.m. are linear, consider an integrated 1-body quantum constraint.
- Algebra generated by minimal set of physically relevant operators (including constraint).

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Setting

GFT with MCMF scalar field

- Free e.o.m.: $\mathcal{D}\varphi \equiv (m^2 + \hbar^2\Delta_g + \lambda\hbar^2\,\partial_\chi^2)\varphi = 0$.
- Quantum constr. $\hat{C} = \int \hat{\varphi}^\dagger \mathcal{D}\hat{\varphi} = m^2\hat{N} - \hat{\Lambda} - \lambda\hat{\Pi}_2$.
- Generators: $\hat{\chi}$, $\hat{\Pi}$, $\hat{\Pi}_2$, $\hat{N}$, $\hat{\Lambda}$ and $\hat{K}$.
- $\hat{K}$ such that $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$.

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Results

Expectation values and variances

- Choose $\hat{K}$ as clock variable.
- Relational evolution of $\langle\hat{\chi}\rangle$ in agreement with classical cosmology.
- Fluctuations are decoupled from expect. values.
- If they are small at small $\langle\hat{K}\rangle$ they stay small even at large $\langle\hat{K}\rangle$ (due to a constant $\langle\hat{N}\rangle$).

LM, Gielen, Oriti, Polaczek 2110.11176.

Inhomogeneous sector

Scalar perturbations from quantum entanglement

Setting

Classical

- 4 MCMF reference fields (χ^0, χ^i),
- 1 MCMF matter field ϕ dominating the energy-momentum budget and slightly relationally inhomogeneous wrt. χ^i.

Jercher, LM, Pithis 2310.17549-2308.13261; LM, Oriti 2112.12677; Jercher, Oriti, Pithis 2206.15442.

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- Geometry from quantum entanglement: inhomogeneities from QG correlations.

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Model

Two-sector GFT

- BC model: $\varphi_\pm \equiv \varphi(g_a, X_\pm, \mathbf{\Phi})$, with $\mathbf{\Phi} = (\chi^\mu, \phi) \in \mathbb{R}^5$ and $K_{\text{GFT}} = K_+ + K_-$
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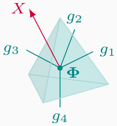
K_+ independent of χ^i. K_- independent of χ^0.



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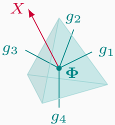

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Collective states

Two-body correlations

notation: $(\cdot, \cdot) = \int_\Omega \mathrm{d}\Omega \cdot \times \cdot$

$$|\Delta\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi})\,|0\rangle$$

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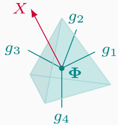

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Background

- $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- τ, σ peaked; $\tilde{\tau}$, $\tilde{\sigma}$ homogeneous.

Jercher, LM, Pithis 2310.17549-2308.13261; LM, Oriti 2112.12677; Jercher, Oriti, Pithis 2206.15442.

Scalar perturbations from quantum entanglement

Setting

Classical

- 4 MCMF reference fields (χ^0, χ^i),
- 1 MCMF matter field ϕ dominating the energy-momentum budget and slightly relationally inhomogeneous wrt. χ^i.

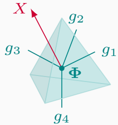
Quantum

- Quanta with spacelike $(+)$ and timelike $(-)$ character to causally couple the physical frame.
- Geometry from quantum entanglement: inhomogeneities from QG correlations.

Model

Two-sector GFT

- BC model: $\varphi_\pm \equiv \varphi(g_a, X_\pm, \mathbf{\Phi})$, with $\mathbf{\Phi} = (\chi^\mu, \phi) \in \mathbb{R}^5$ and $K_{\mathrm{GFT}} = K_+ + K_-$
- Since χ^0 (χ^i) propagates along timelike (spacelike) edges:



K_+ independent of χ^i. K_- independent of χ^0.

Collective states

Two-body correlations

notation: $(\cdot\,,\,\cdot) = \int_\Omega \mathrm{d}\Omega \,\cdot \times \cdot$

$$|\Delta\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi})\,|0\rangle$$

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Perturbations

- $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- $\delta\Phi$, $\delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- Pert. = rel. nearest neighbour 2-body correlations.

Jercher, LM, Pithis 2310.17549-2308.13261; LM, Oriti 2112.12677; Jercher, Oriti, Pithis 2206.15442.

E.o.m.

Mean-field dynamics

- 2 mean-field eqs. for 3 variables $(\delta\Phi, \delta\Psi, \delta\Xi)$:

$$\langle \delta S/\delta\hat{\varphi}_+^\dagger \rangle_\Delta = 0 = \langle \delta S/\delta\hat{\varphi}_-^\dagger \rangle_\Delta$$

- Late times and single (spacelike) rep. label.

Jercher, LM, Pithis 2310.17549-2308.13261.

Emergent dynamics of cosmic inhomogeneities

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Effective dynamics

Classical dynamics with trans-Planckian QG effects

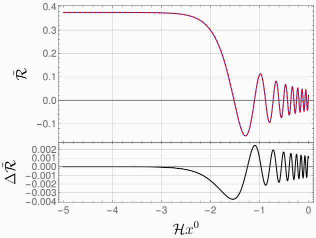
- Scalar (isotropic) perturbations dynamics from dynamics of QG correlations $(\delta\Phi, \delta\Psi, \delta\Xi)$.
- E.g.: matter $\delta\phi_{\mathrm{GFT}}$ and "curvature-like" $\tilde{\mathcal{R}}$:

$$\delta\phi''_{\mathrm{GFT}} + k^2 a^4 \delta\phi_{\mathrm{GFT}} = \left(\frac{a^2 k}{M_{\mathrm{pl}}}\right) j_\phi[\bar{\phi}]\,,$$

$$\tilde{\mathcal{R}}''_{\mathrm{GFT}} + k^2 a^4 \tilde{\mathcal{R}}_{\mathrm{GFT}} = \left(\frac{a^2 k}{M_{\mathrm{pl}}}\right) j_{\tilde{\mathcal{R}}}[\bar{\phi}]\,,$$

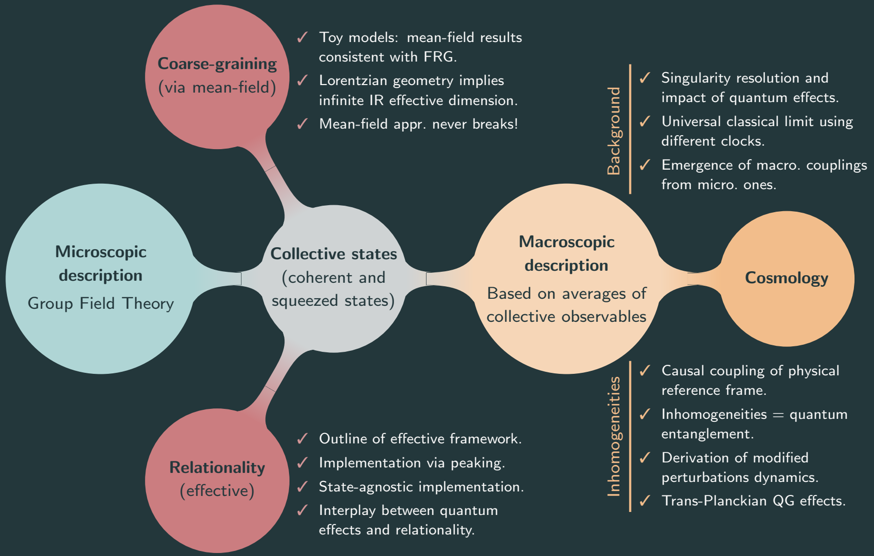
- Trans-Planckian QG corrections to the dynamics of scalar isotropic perturbations.

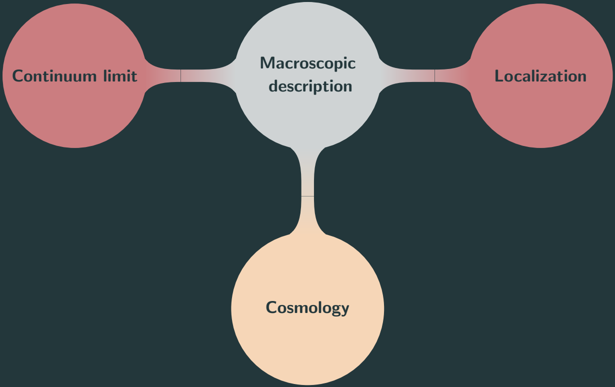
✓ Remarkable agreement with GR at larger scales.

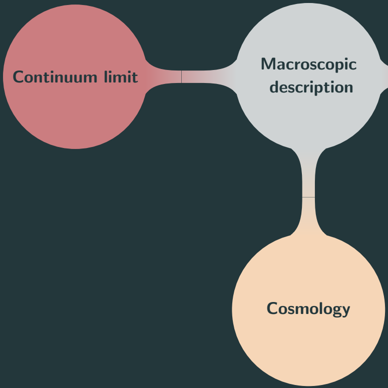


Top: $\tilde{\mathcal{R}}_{\mathrm{GFT}}$ (blue) and $\tilde{\mathcal{R}}_{\mathrm{GR}}$ (dashed red) for $k/M_{\mathrm{Pl}} = 10^2$. Bottom: their difference $\Delta\tilde{\mathcal{R}}$.

Jercher, LM, Pithis 2310.17549-2308.13261.


Coarse-graining
(via mean-field)
✓ Toy models: mean-field results consistent with FRG.
✓ Lorentzian geometry implies infinite IR effective dimension.
✓ Mean-field appr. never breaks!
Microscopic description
Group Field Theory
Collective states
(coherent and squeezed states)
Macroscopic description
Based on averages of collective observables
Cosmology
Background
✓ Singularity resolution and impact of quantum effects.
✓ Universal classical limit using different clocks.
✓ Emergence of macro. couplings from micro. ones.
Inhomogeneities
✓ Causal coupling of physical reference frame.
✓ Inhomogeneities = quantum entanglement.
✓ Derivation of modified perturbations dynamics.
✓ Trans-Planckian QG effects.
Relationality
(effective)
✓ Outline of effective framework.
✓ Implementation via peaking.
✓ State-agnostic implementation.
✓ Interplay between quantum effects and relationality.


Macroscopic description
Continuum limit
Localization
Cosmology



- Effective state-agnostic approach:
  - Extension to gauge theories: quantum covariant phase space using nPI action.
  - Background independent theories.
  - Boundaries, QRFs and edge-modes.
- Relational RG scheme:
  - Relational scale in asymptotic safety.
- Relational observables in GFT using POVMs.

Continuum limit — **Macroscopic description** — **Localization**

Cosmology

- Construct effective GFT models.
- Renormalization of geometric models:
  - Identify equivalent tensor models.
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- Connect with spin foams and LQG:
  - Spin foam TNR and refinement limit.
  - LQG-Spin foam-GFT map in HK model.
  - Compare LQG&GFT averaged dynamics.

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Cosmology

- Cosmic acceleration from QG:
  - Slow-roll inflation from GFT interactions.
  - Early dark energy? H_0 tension?
  - Constraints on GFT models?
- QG and cosmological perturbations:
  - Phenom. implementation of QG effects on SCM; comparison with observations.
  - Full cosmological perturbation theory from GFTs: more observables, realistic matter, primordial power spectrum.
- Near-bounce cosmic dynamics:
  - Mismatch of super-horizon dynamics with MG.
  - Suppression/enhancement of chaotic behavior?

Backup

Specifics of GFT models

EPRL model

$$S=\sum_{\{j_a\},\{j'_a\},\{m_a\},\{m'_a\},\iota,\iota'}\bar{\varphi}^{\{j_a\}\iota}_{\{m_a\}}\varphi^{\{j'_a\}\iota'}_{\{m'_a\}}\mathcal{K}^{\{j_a\}\{j'_a\}\iota\iota'}_{\{m_a\}\{m'_a\}}+V_5\,,$$

$$V_5=\frac{1}{5}\sum_{\{j_a\},\{m_a\},\{\iota_b\}}\varphi^{j_1j_2j_3j_4\iota_1}_{m_1m_2m_3m_4}\varphi^{j_4j_5j_6j_7\iota_2}_{-m_4m_5m_6m_7}\varphi^{j_7j_3j_8j_9\iota_3}_{-m_7-m_3m_8m_9}\varphi^{j_9j_6j_2j_{10}\iota_4}_{-m_9-m_6-m_2m_{10}}\varphi^{j_{10}j_8j_5j_1\iota_5}_{-m_{10}-m_8-m_5-m_1}$$

$$\times\prod_{c=1}^{10}(-1)^{j_c-m_c}\mathcal{V}_5(j_1,\dots,j_{10};\iota_1,\dots,\iota_5)\,,$$

$$\begin{aligned}a&=1,\dots,4\\ b&=1,\dots,5\\ c&=1,\dots,10\end{aligned}$$

$$\mathcal{V}_5(\{j_c\},\{\iota_b\})=\sum_{\{n_A\}}\int\left[\prod_A\mathrm{d}\rho_A(n_A^2+\rho_A^2)\right]\left[\bigotimes_b f^{\iota_b}_{\{n_A\}\{\rho_A\}}(\{j_a\})\right]\{15j\}_{\mathrm{SL}(2,\mathbb{C})}\,,$$

where f maps $\mathrm{SL}(2,\mathbb{C})$ data into $\mathrm{SU}(2)$ ones by imposing the constraints $n=2j$ and $\rho=2j\gamma$.

Extended BC model

$$S=\left[\prod_i\int\mathrm{d}\rho_i\,4\rho_i^2\sum_{j_im_i}\right]\bar{\varphi}^{\rho_i}_{j_im_i}\varphi^{\rho_i}_{j_im_i}+\frac{\lambda}{5}\left[\prod_{a=1}^{10}\int\mathrm{d}\rho_a\,4\rho_a^2\sum_{j_am_a}\right]\left[\prod_{a=1}^{10}(-1)^{-j_a-m_a}\right]\{10\rho\}_{\mathrm{BC}}$$

$$\times\,\varphi^{\rho_1\rho_2\rho_3\rho_4}_{j_1m_1j_2m_2j_3m_3j_4m_4}\varphi^{\rho_4\rho_5\rho_6\rho_7}_{j_4-m_4j_5m_5j_6m_6j_7m_7}\varphi^{\rho_7\rho_3\rho_8\rho_9}_{j_7-m_7j_3-m_3j_8m_8j_9m_9}$$

$$\times\,\varphi^{\rho_9\rho_6\rho_2\rho_{10}}_{j_9-m_9j_6-m_6j_2-m_2j_{10}m_{10}}\varphi^{\rho_{10}\rho_8\rho_5\rho_1}_{j_{10}-m_{10}j_8-m_8j_5-m_5j_1-m_1}+\text{c.c.}$$

Engle, Livine, Pereira, Rovelli 0711.0146; Gielen, Oriti, Sindoni 1311.1238; Jercher, Oriti, Pithis 2112.00091.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:
σ depends on a MCMF "clock" scalar field χ^0

LM, Oriti 2008.02774 ; LM, Oriti 2010.09700.

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Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot,\cdot) = \int \mathrm{d}\chi^0 \mathrm{d}g_a$):

$$\hat{X}^0 = \left(\hat{\varphi}^\dagger, \chi^0 \hat{\varphi}\right) \qquad \hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi}) \qquad \hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

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$\xrightarrow[\textbf{isotropy}]{\text{wavefunction}}$

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_j V_j |\tilde{\sigma}_j|^2(x^0)$$

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Relationality

Clock expectation values

For large N, x^0 has a clear physical meaning:

$$\langle \hat{\chi}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / N \qquad \textit{(intensive)}$$

$$= x^0 \left(1 + \delta X(x^0)/N(x^0)\right)$$

$$\langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}} = \langle \hat{H}_\sigma \rangle_{\sigma_{x^0}} \left(1 + \text{const.}/N(x^0)\right)$$

Clock variances

For large N, clock fluctuations scale as N^{-1}:

$$\Delta^2_{\sigma_{x^0}} \chi^0 < \frac{1}{N}\left(1 + \frac{\epsilon}{2(x^0)^2} \frac{1}{(1+\delta X/N)^2}\right)$$

$$\Delta^2_{\sigma_{x^0}} \Pi^0 = \Delta^2_{\sigma_{x^0}} H_\sigma \left(1 + \text{const.}/N(x^0)\right)$$

$$\Delta^2_{\sigma_{x^0}} H_\sigma = \Delta^2_{\sigma_{x^0}} N = N^{-1}(x^0)\,.$$

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Clock POVMs

There cannot exist a self-adjoint (monotonic) $\hat{T}$ canonically conjugate to a bounded $\hat{H}_C$.

LM, Oriti, Wilson-Ewing (in progress).

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A POVM $\hat{E}_T : \mathcal{B}(G) \to \mathcal{L}_B(\mathcal{H})$ satisfies

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Relational strategy in full GFT

Quantum Mechanics

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Group Field Theory

Scalar field clock POVMs

$$\hat{E}_\chi = |0\rangle\langle 0| + \mathrm{d}\chi \sum_{n=1}^{\infty} \frac{1}{n!} \int \left[\prod_{i=1}^{n} \mathrm{d}\chi_i \, \mathrm{d}\xi_i\right] \frac{\sum_{i=1}^{n} \delta(\chi_i - \chi)}{n} \left[\prod_{i=1}^{n} \hat{\varphi}^\dagger(\chi_i, \xi_i)\right] |0\rangle\langle 0| \left[\prod_{i=1}^{n} \hat{\varphi}(\chi_i, \xi_i)\right]$$

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$\hat{E}_\chi$ represents a scalar field measurement

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$\hat{E}_\chi$ represents a scalar field measurement

Relational observables

P.W.-like: $\langle \hat{\Xi}_\chi \rangle_\psi \propto \langle \{\hat{\Xi}, \hat{E}_\chi\} \rangle_\psi$

- Is it a sensible definition? $\hat{E}_\chi$ is not a projector!
- ⚠ Compare with previous results when $|\psi\rangle = |\sigma\rangle$!

LM, Oriti, Wilson-Ewing (in progress).