Relational physics in Group Field Theories

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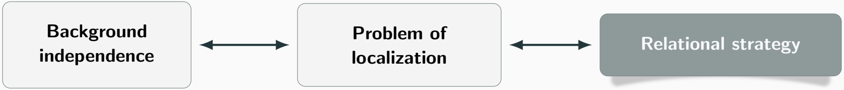
Overview

- **The relational strategy**
  - The classical and quantum perspectives
  - The emergent perspective and effective methods
  - Relational strategy and GFTs
- **Effective approaches**
  - General considerations
  - Coherent Peaked States
- **Conclusions**

The relational strategy

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Isham 9210011; Rovelli Class. Quantum Grav. 8 297; Dittrich 0507106; Tambornino 1109.0740; Giesel, Thiemann 0711.0119 . . .

Relational strategy: the classical and quantum GR perspective

Background independence ⟷ **Problem of localization** ⟷ **Relational strategy**

Quite well understood from a classical perspective, less from a quantum perspective.

Classical

Physical localization via relational observables:

- Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ.
- Evolution in τ is relational.
- $F_{f,T}(\tau)$ is a very complicated function.
- Applications almost only for very simple systems.

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Quantum GR

Dirac approach: Quantize first.

- Perspective neutral.
- Poor control of the physical Hilbert space.

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Reduced approach: Relationality first.

- No quantum constraint to solve.
- Not perspective neutral. Too complicated to implement in most of the cases.

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Relational strategy and emergent quantum gravity theories

Background independence ⟷ **Problem of localization** ⟷ **Relational strategy**

A genuinely new dimension of the problem arises for emergent QG theories.

LM, Oriti 2008.02774; Giulini 0603087; Kuchar Int.J.Mod.Phys.D 20(2011); Isham 9210011; Rovelli Class. Quantum Grav. 8 297;

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Microscopic pre-geo

- Fundamental d.o.f. are weakly related to spacetime quantities;
- The latter expected to emerge from the former in an appropriate phase.

Macroscopic proto-geo

- Set of collective observables;
- Coarse grained states or probability distributions.

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Effective approaches:

- More mathematical control and physical insights.
- Relevant for observative purposes.

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Relational strategy and the GFT Fock space

Relational strategy in GFT: difficulties

Ante quantum

Classical

- N classical GFT atoms: $\mathcal{C}^{(i)} = G^d \times \mathbb{R}^{d_l}$.
- ith-atom deparametrizable wrt. a clock $\chi^{0,(i)}$.
- Synchronize the clocks $\chi^{0,(i)} \longrightarrow t_N$.
- Deparametrized N-atoms system: $\mathcal{C}_N = \mathbb{R} \times \Gamma_N$.

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Quantum

- $\mathcal{F}_{\text{red}} = \bigoplus_N \text{sym}\mathcal{H}_N$, generated by $(\varphi^\dagger, |0\rangle)$.
- But φ, $\varphi^\dagger$ satisfy equal-time (t_F) CCR!

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Relational observables?

How to construct them without having manifest access to diffeos?

↓

Simplest ansatz: localize operators wrt. clock data.

$$\hat{N} = \int \mathrm{d}g_a \, \mathrm{d}\chi \, \hat{\varphi}^\dagger(g_a, \chi) \hat{\varphi}(g_a, \chi) \,,$$

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A scalar field should be represented as an operator on $\mathcal{F}_{\text{GFT}}$.

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$\chi = \hat{\chi}$-eigenvalue on "synchronous" states.

- How to properly define $\hat{\chi}$?
- Extension to generic states and operators?

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What is relational time in $\mathcal{F}_{\text{GFT}}$?

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Relational strategy in GFT: difficulties

The pre-geometric, many-body nature of GFTs hinders the implementation of the relational strategy!

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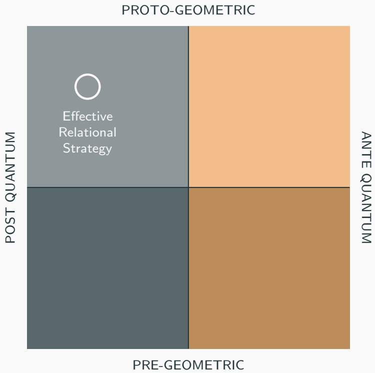
Effective approaches

Emergent effective relational strategy



Basic principles

Emergence Relational strategy in terms of collective observables and states.

Effectiveness Averaged relational localization. Internal frame not too quantum.

LM, Oriti 2008.02774; Bojowald, Hoehn, Tsobanjan 1011.3040; Bojowald, Tsobanjan 0906.1772;

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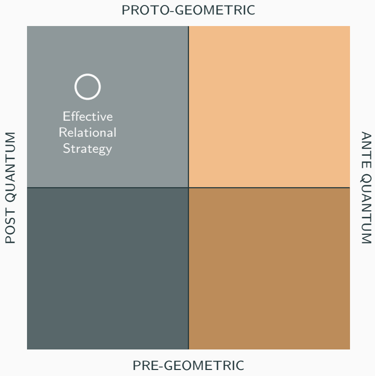

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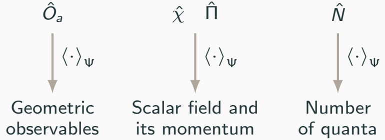
Concrete example: scalar field clock

Emergence

- Identify (collective) states $|\Psi\rangle$ admitting a continuum proto-geometric interpretation.
- Identify a set of collective observables:



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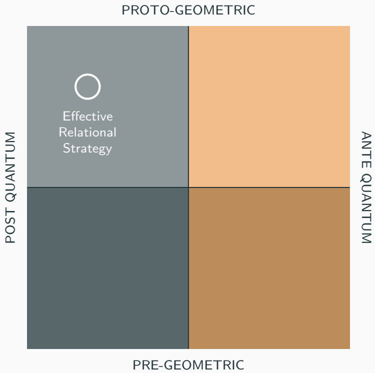

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Emergence

- Identify (collective) states $|\Psi\rangle$ admitting a continuum proto-geometric interpretation.
- Identify a set of collective observables:

$\hat{O}_a$ $\xrightarrow{\langle\cdot\rangle_\Psi}$ Geometric observables

$\hat{\chi}$ $\hat{\Pi}$ $\xrightarrow{\langle\cdot\rangle_\Psi}$ Scalar field and its momentum

$\hat{N}$ $\xrightarrow{\langle\cdot\rangle_\Psi}$ Number of quanta

Effectivness

- It exists a "Hamiltonian" $\hat{H}$ such that

$$i\frac{\mathrm{d}}{\mathrm{d}\,\langle\hat{\chi}\rangle_\Psi}\langle\hat{O}_a\rangle_\Psi = \langle[\hat{H},\hat{O}_a]\rangle_\Psi\ ,$$

  and whose moments coincide with those of $\hat{\Pi}$.
- Relative fluctuations of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$:

$$\Delta^2\chi \ll 1\,, \qquad \Delta^2\chi \sim \langle\hat{N}\rangle_\Psi^{-1}\ .$$

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Coherent Peaked States

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Collective states

GFT coherent states

- From the GFT perspective, continuum geometries are associated to large number of quanta.
- The simplest states that can accommodate infinite number of quanta are coherent states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp\left[\int \mathrm{d}^{d_l}\chi \int \mathrm{d}g_a\, \sigma(g_a, \chi^\alpha)\hat{\varphi}^\dagger(g_a, \chi^\alpha)\right] |0\rangle \,.$$

- Assuming $\sigma(g_a, \cdot) = \sigma(hg_a h', \cdot)$: $\mathcal{D}$ = space of spatial geometries + matter at a point.
- Dynamics of σ determined by mean-field equations ("hydrodynamic approximation").

Relationality

Coherent Peaked States

- Constructing relational observables on $\mathcal{F}_{\mathrm{GFT}}$ is difficult (QFT with no continuum intuition).
- Relational localization implemented at an effective level on observable averages.

LM, Oriti 2008.02774; LM, Oriti 2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238; Gielen 1404.2944.

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- If χ^μ constitute a reference frame, this can be achieved by assuming

$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})\,.$$

Peaking cannot be perfect to avoid large clock momentum fluctuations (Heisenberg principle).

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  Peaking cannot be perfect to avoid large clock momentum fluctuations (Heisenberg principle).
- σ is a distribution of (discrete) spatial geometries and matter at points x^μ in the frame manifold.

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Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:
σ depends on a MCMF "clock" scalar field χ^0

LM, Oriti 2008.02774 ; LM, Oriti 2010.09700.

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Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot,\cdot) = \int \mathrm{d}\chi^0 \mathrm{d}g_a$):

$$\hat{X}^0 = \left(\hat{\varphi}^\dagger, \chi^0 \hat{\varphi}\right) \qquad \hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

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$\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0 = x^0}$: functionals of $\tilde{\sigma}$ localized at x^0

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wavefunction → **isotropy**

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_j V_j |\tilde{\sigma}_j|^2 (x^0)$$

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Relationality

Clock expectation values

For large N, x^0 has a clear physical meaning:

$$\langle \hat{\chi}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / N \qquad \textit{(intensive)}$$

$$= x^0 \left(1 + \delta X(x^0)/N(x^0)\right)$$

$$\langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}} = \langle \hat{H}_\sigma \rangle_{\sigma_{x^0}} \left(1 + \text{const.}/N(x^0)\right)$$

Clock variances

For large N, clock fluctuations scale as N^{-1}:

$$\Delta^2_{\sigma_{x^0}} \chi^0 < \frac{1}{N}\left(1 + \frac{\epsilon}{2(x^0)^2} \frac{1}{(1+\delta X/N)^2}\right)$$

$$\Delta^2_{\sigma_{x^0}} \Pi^0 = \Delta^2_{\sigma_{x^0}} H_\sigma \left(1 + \text{const.}/N(x^0)\right)$$

$$\Delta^2_{\sigma_{x^0}} H_\sigma = \Delta^2_{\sigma_{x^0}} N = N^{-1}(x^0)\,.$$

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Classical limit and validity of the framework

Effective dynamics

Mean-field approximation

- Mesoscopic regime: large N but negligible interactions.
- Derivative expansion of $\mathcal{K}$ (due to peaking properties).
- Isotropy: $\tilde{\sigma}_j \equiv \rho_j e^{i\theta_j}$ fundamental variables.

$$\tilde{\sigma}_j'' - 2i\tilde{\pi}_0\tilde{\sigma}_j' - E_j^2\tilde{\sigma} = 0.$$

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Effective volume dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j V_j\rho_j \mathrm{sgn}(\rho_j')\sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2\sum_j V_j\left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}$$

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$$\tilde{\sigma}_j'' - 2i\tilde{\pi}_0\tilde{\sigma}_j' - E_j^2\tilde{\sigma} = 0.$$

Effective volume dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j V_j\rho_j \mathrm{sgn}(\rho_j')\sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2\sum_j V_j\left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}$$

Classical limit

Large number of quanta (large volume and late times)

- ✓ Volume quantum fluctuations under control.
- If μ_j^2 is mildly dependent on j (or one j is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881.

Classical limit and validity of the framework

Effective dynamics

Mean-field approximation

- Mesoscopic regime: large N but negligible interactions.
- Derivative expansion of $\mathcal{K}$ (due to peaking properties).
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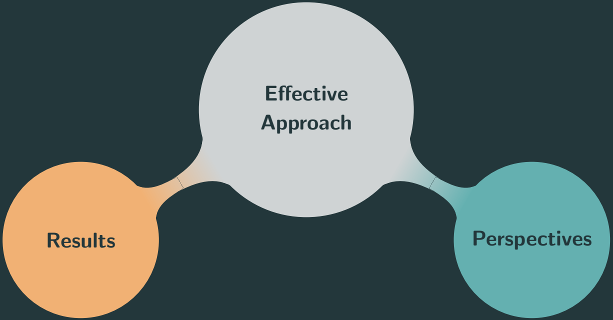
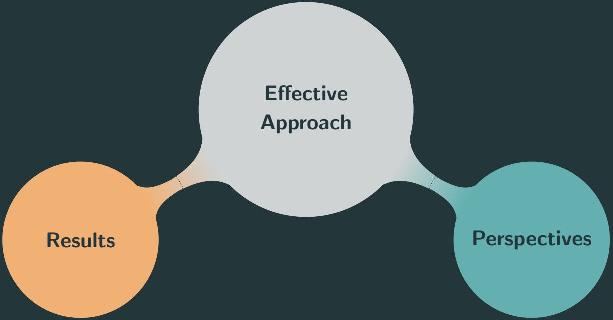
- ✓ $x^0 = \langle \hat{\chi}^0 \rangle_{\sigma_{x^0}}$.
- ✓ Clock quantum fluctuations negligible.
- ✓ $\langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}} = \langle \hat{H}_\sigma \rangle_{\sigma_{x^0}}$ (higher moments $\simeq 0$).

Effective relational framework reliable!

LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881.

Conclusions


Effective Approach
Results
Perspectives



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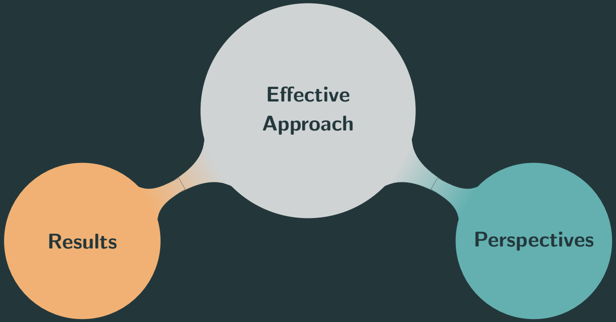

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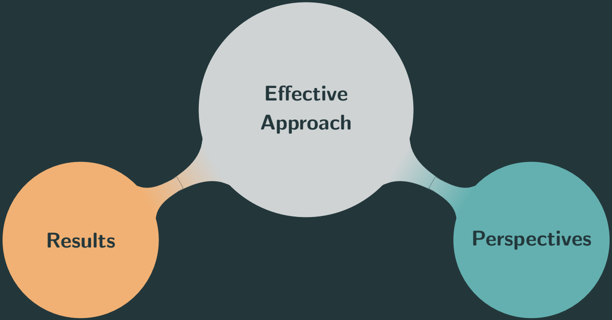

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What about relational observables in full GFT?

GFTs: pre-geometric many-body


No explicit notion of diffeos
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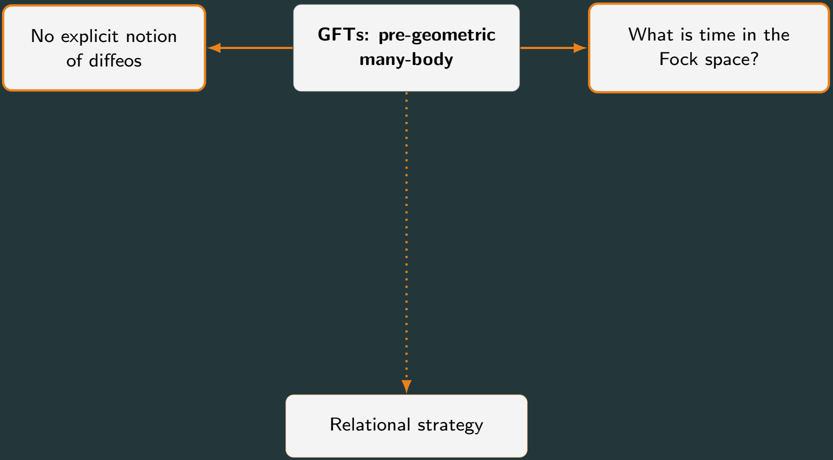
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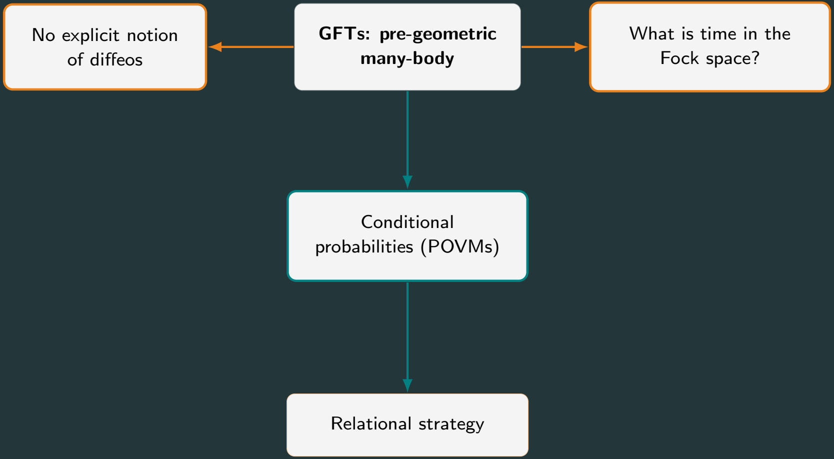
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Conditional probabilities (POVMs)
Relational strategy

Clock POVMs

There cannot exist a self-adjoint (monotonic) $\hat{T}$ canonically conjugate to a bounded $\hat{H}_C$.

LM, Oriti, Wilson-Ewing (in progress).

Relational strategy in full GFT

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A POVM $\hat{E}_T : \mathcal{B}(G) \to \mathcal{L}_B(\mathcal{H})$ satisfies

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Group Field Theory

Scalar field clock POVMs

$$\hat{E}_\chi = |0\rangle\langle 0| + \mathrm{d}\chi \sum_{n=1}^{\infty} \frac{1}{n!} \int \left[\prod_{i=1}^{n} \mathrm{d}\chi_i \, \mathrm{d}\xi_i\right] \frac{\sum_{i=1}^{n} \delta(\chi_i - \chi)}{n} \left[\prod_{i=1}^{n} \hat{\varphi}^\dagger(\chi_i, \xi_i)\right] |0\rangle\langle 0| \left[\prod_{i=1}^{n} \hat{\varphi}(\chi_i, \xi_i)\right]$$

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$\hat{E}_\chi$ represents a scalar field measurement

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$$\left\langle \hat{O}_\chi^{(\chi,\xi)} \right\rangle_\psi = \left\langle \{\hat{O}_\chi^{(\chi,\xi)}, \hat{E}_\chi\} \right\rangle_\psi$$

- Is it a sensible definition? $\hat{E}_\chi$ is not a projector!
- ⚠ Compare with previous results when $|\psi\rangle = |\sigma\rangle$!

LM, Oriti, Wilson-Ewing (in progress).

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U(1) example

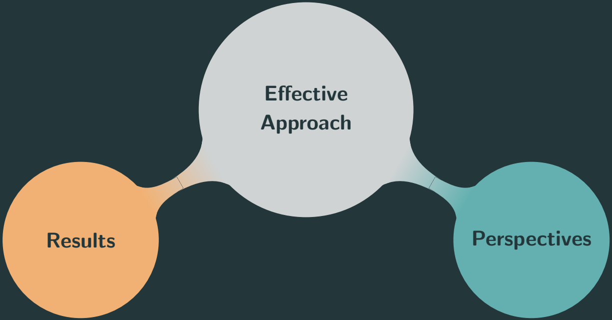
Comparison with previous results

$$\left\langle \hat{O}_\chi^{(x,\xi)} \right\rangle_\sigma = \mathrm{d}\chi \left\{ \frac{1 - |\mathcal{N}|^2}{\|\sigma\|} \left[O^{(x,\xi)}(\chi) - \langle \hat{O}^{(x,\xi)} \rangle_\sigma \frac{N(\chi)}{\|\sigma\|} \right] + \langle \hat{O}^{(x,\xi)} \rangle_\sigma \frac{N(\chi)}{\|\sigma\|} \right\}.$$

- $\mathcal{N}$ normalization; $\|\sigma\|$ condensate norm;
- $\hat{O}^{(x,\xi)}$ perspective-neutral obs.;
- $O^{(x,\xi)}(\chi)$ exp. values of deparametrized obs.;
- When $\|\sigma\| \ll 1$, $\langle \hat{O}_\chi^{(x,\xi)} \rangle_\sigma = \mathrm{d}\chi O^{(x,\xi)}(\chi)$.

- ✓ Number: $\langle \hat{N}_\chi \rangle_\sigma = \mathrm{d}\chi N(\chi)$.
- ✓ Scalar field: $\langle \hat{X}_\chi \rangle_\sigma / N(\chi) \propto \mathrm{d}\chi \chi$.
- ✓ Geometry: $\langle \hat{O}_\chi^{(\xi)} \rangle_\sigma \propto \mathrm{d}\chi O^{(\xi)}(\chi)$ when $\|\sigma\| \gg 1$ and single mode (= if symmetric).

LM, Oriti, Wilson-Ewing (in progress).



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Backup

Effective approach for constrained quantum systems

How does our scheme for extraction of relational cosmological physics depend on the specific choice of states? ⟷ **A “state-agnostic” strategy is needed!**

LM, Gielen, Oriti, Polaczek 2110.11176; Bojowald, Sandhoefer, Skirzewski, Tsobanjan 0804.3365; Bojowald Tsobanjan 0906.1772.

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Effective state-agnostic approach for constrained quantum systems

Construction of the effective system

Step 1: definition of the quantum phase space

- Describe the system with exp. values $\langle \hat{A}_i \rangle$ and moments:
- Poisson structure inherited from the algebra structure

$$\left\{ \langle \hat{A}_i \rangle , \langle \hat{A}_j \rangle \right\} = (i\hbar)^{-1} \left\langle [\hat{A}_i, \hat{A}_j] \right\rangle \text{ (same for } \Delta\text{s).}$$

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- Generate gauge transf. on the quantum phase space.

Step 3: truncation scheme (e.g. semiclassicality)

Relational description

Step 1: choose a clock $\hat{T}$ ($[\hat{T}, \hat{P}]$ closes)

Step 2: gauge fixing

- At 1st order: $\Delta(TA_i) = 0$, $A_i \in \mathcal{A} \backslash \{\hat{P}\}$.
- Use constraints to eliminate $\hat{P}$-variables.

LM, Gielen, Oriti, Polaczek 2110.11176; Bojowald, Sandhoefer, Skirzewski, Tsobanjan 0804.3365; Bojowald Tsobanjan 0906.1772.

Effective approach for constrained quantum systems

How does our scheme for extraction of relational cosmological physics depend on the specific choice of states? ⟷ **A "state-agnostic" strategy is needed!**

Effective state-agnostic approach for constrained quantum systems

Construction of the effective system

Step 1: definition of the quantum phase space

- Describe the system with exp. values $\langle \hat{A}_i \rangle$ and moments:
- Poisson structure inherited from the algebra structure

$$\left\{ \langle \hat{A}_i \rangle , \langle \hat{A}_j \rangle \right\} = (i\hbar)^{-1} \left\langle [\hat{A}_i, \hat{A}_j] \right\rangle \text{ (same for } \Delta\text{s).}$$

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Step 3: relational rewriting

- Determine the remaining gauge flow which preserves the gauge conditions.
- Write evolution of the remaining variables wrt. T (classical clock).

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A state agnostic approach: application to GFT

How can this framework be generalized to a **field theory context**?

Infinitely many algebra generators.

Infinitely many quantum constraints.

LM, Gielen, Oriti, Polaczek 2110.11176.

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Motivations

- Interest in a coarse grained system characterized by a small number of macroscopic (1-body) observables.
- Expected to be the case for cosmology.

Coarse-graining truncation

- When the e.o.m. are linear, consider an integrated 1-body quantum constraint.
- Algebra generated by minimal set of physically relevant operators (including constraint).

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Setting

GFT with MCMF scalar field

- Free e.o.m.: $\mathcal{D}\varphi \equiv (m^2 + \hbar^2\Delta_g + \lambda\hbar^2\,\partial^2_\chi)\varphi = 0$.
- Quantum constr. $\hat{C} = \int \hat{\varphi}^\dagger \mathcal{D}\hat{\varphi} = m^2\hat{N} - \hat{\Lambda} - \lambda\hat{\Pi}_2$.
- Generators: $\hat{X}$, $\hat{\Pi}$, $\hat{\Pi}_2$, $\hat{N}$, $\hat{\Lambda}$ and $\hat{K}$.
- $\hat{K}$ such that $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$.

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Results

Expectation values and variances

- The procedure can naturally be carried over by choosing as clock variable $\hat{K}$.
- Relational evolution of $\langle\hat{X}\rangle$ in agreement with classical cosmology.
- Fluctuations are decoupled from expect. values.
- If they are small at small $\langle\hat{K}\rangle$ they stay small even at large $\langle\hat{K}\rangle$ (probably associated to a constant $\langle\hat{N}\rangle$).

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