

Scalar cosmological perturbations from full quantum gravity

In collaboration with: D. Oriti, E. Wilson-Ewing, A. Pithis, A. Jercher, P. Fischer

Luca Marchetti

Quantum Gravity 2023

Radboud University

14 July 2023

Department of Mathematics and Statistics

UNB Fredericton

The diagram consists of two main parts on a dark gray background. On the left is a light blue circle containing the text 'Microscopic description' and 'Background independent, pre-geometric'. On the right is a light orange shape composed of two circles of different sizes joined at a narrow neck. The larger circle on the left contains the text 'Macroscopic description', and the smaller circle on the right contains the text 'Continuum physics'.

**Microscopic
description**

Background independent,
pre-geometric

**Macroscopic
description**

**Continuum
physics**

**Localization
problem**

**Microscopic
description**

Background independent,
pre-geometric

**Continuum limit
problem**

**Macroscopic
description**

**Continuum
physics**

The diagram consists of two main groups of circles on a dark gray background. On the left, three light blue circles are arranged in a vertical column. The top circle is labeled 'Relationality'. The middle circle is labeled 'Microscopic description' and contains the text 'Background independent, pre-geometric'. The bottom circle is labeled 'Coarse-graining'. On the right, two orange circles are connected by a horizontal bridge. The left circle is labeled 'Macroscopic description' and the right circle is labeled 'Continuum physics'.

Relationality

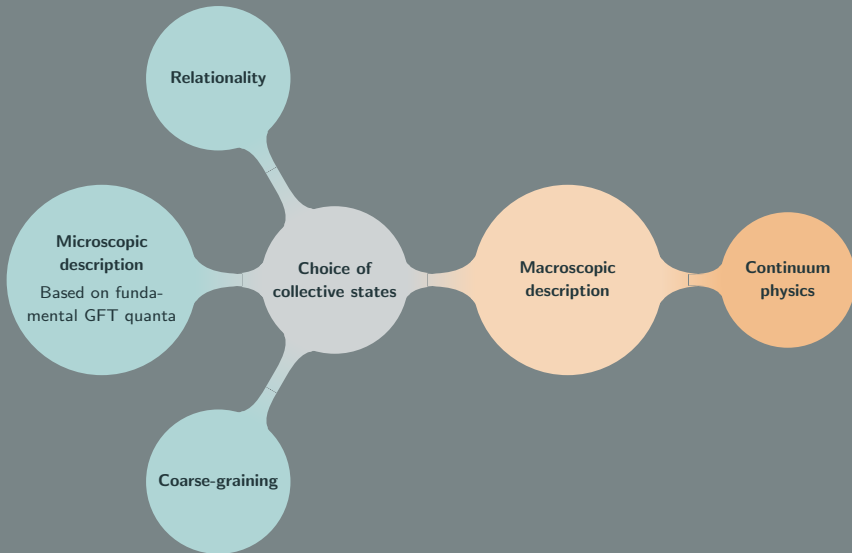
**Microscopic
description**

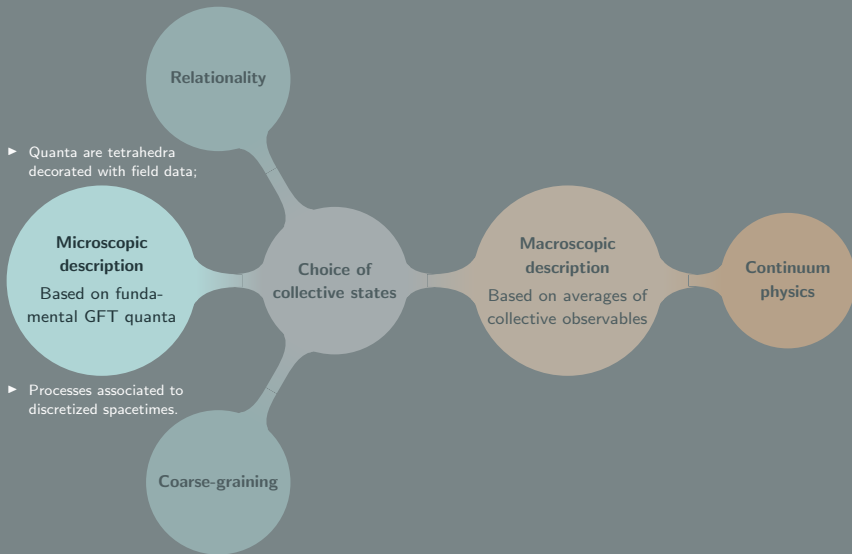
Background independent,
pre-geometric

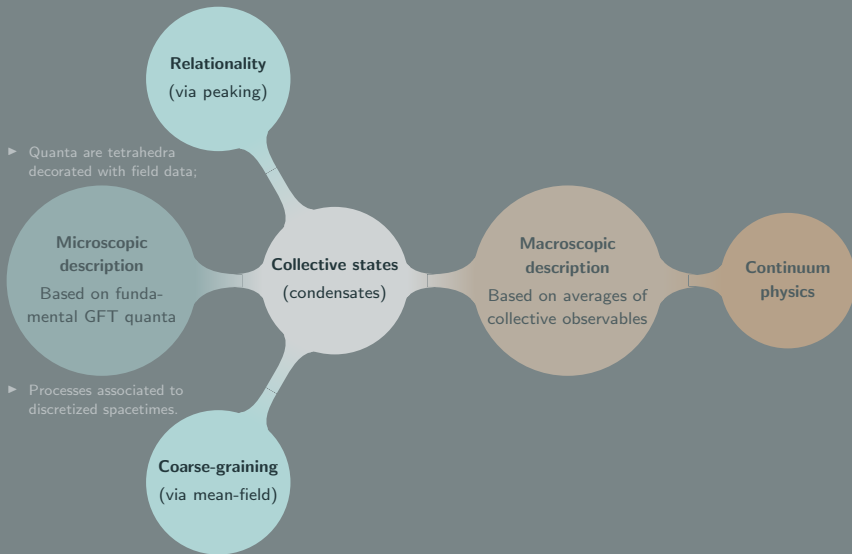
Coarse-graining

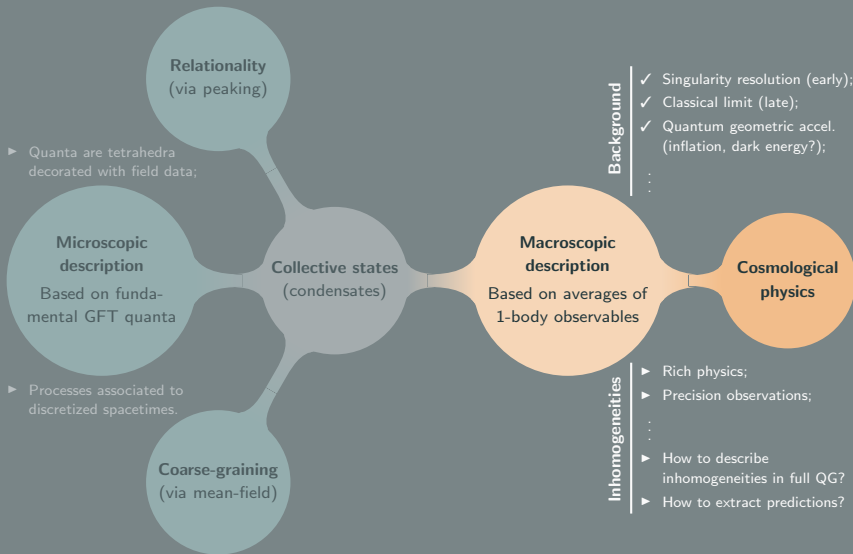
**Macroscopic
description**

**Continuum
physics**









Inhomogeneities in GFT cosmology

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Quantum

- ▶ $\hat{\varphi}(g_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} from EPRL-like model.

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Quantum

- ▶ $\hat{\varphi}(g_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} from EPRL-like model.

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi dg_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

Mat. Vol. Frame

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi dg_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

Quantum

- ▶ $\hat{\varphi}(g_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} from EPRL-like model.

States

- ▶ Peaked $|\sigma\rangle_x$ around $\chi^\mu = x^\mu$, with $\sigma = \eta \times \tilde{\sigma}$:
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- Peaked $|\sigma\rangle_x$ around $\chi^\mu = x^\mu$, with $\sigma = \eta \times \tilde{\sigma}$:
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Late times volume and matter dynamics

- $\langle \delta S / \delta \hat{\varphi} \rangle_{\sigma_x} = 0$ (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- Decoupling for a range of values of $|\sigma\rangle_x$ and large N (**late times**).

single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_x}, \langle \hat{\Phi} \rangle_{\sigma_x}$.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- Peaked $|\sigma\rangle_x$ around $\chi^\mu = x^\mu$, with $\sigma = \eta \times \tilde{\sigma}$:
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Late times volume and matter dynamics

- $\langle \delta S / \delta \hat{\varphi} \rangle_{\sigma_x} = 0$ (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- Decoupling for a range of values of $|\sigma\rangle_x$ and large N (**late times**).

single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_x}, \langle \hat{\Phi} \rangle_{\sigma_x}$.

Background

- ✓ Matching with GR possible.
- Macro. couplings defined in terms of GFT ones.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ Peaked $|\sigma\rangle_x$ around $\chi^\mu = x^\mu$, with $\sigma = \eta \times \tilde{\sigma}$:
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Late times volume and matter dynamics

- ▶ $\langle \delta S / \delta \hat{\varphi} \rangle_{\sigma_x} = 0$ (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- ▶ Decoupling for a range of values of $|\sigma\rangle_x$ and large N (**late times**).

single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_x}, \langle \hat{\Phi} \rangle_{\sigma_x}$.

Background

- ✓ Matching with GR possible.
- ▶ Macro. couplings defined in terms of GFT ones.

Perturbations

- ✓ “Deep super-horizon” ($k \rightarrow 0$) GR matching.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ Peaked $|\sigma\rangle_x$ around $\chi^\mu = x^\mu$, with $\sigma = \eta \times \tilde{\sigma}$:
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Late times volume and matter dynamics

- ▶ $\langle \delta S / \delta \hat{\varphi} \rangle_{\sigma_x} = 0$ (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
 - ▶ Decoupling for a range of values of $|\sigma\rangle_x$ and large N (**late times**).
- $\xrightarrow[\text{label}]{\text{single}}$ Dynamic equations for $\langle \hat{V} \rangle_{\sigma_x}, \langle \hat{\Phi} \rangle_{\sigma_x}$.

Background

- ✓ Matching with GR possible.
- ▶ Macro. couplings defined in terms of GFT ones.

Perturbations

- ✓ “Deep super-horizon” ($k \rightarrow 0$) GR matching.
- ▶ **Unphysical behavior** of spatial derivative terms.

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Deep super-horizon volume and matter dynamics

- Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- Restrict to deep super-horizon modes but at **early times**.

single \longrightarrow spin \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}, \langle \hat{\Phi} \rangle_{\sigma_{x^0}}$

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Deep super-horizon volume and matter dynamics

- Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- Restrict to deep super-horizon modes but at **early times**.

single \longrightarrow Dynamic equations
spin for $\langle \hat{V} \rangle_{\sigma_{x^0}}, \langle \hat{\Phi} \rangle_{\sigma_{x^0}}$

Modified gravity

- Dynamics of deep super-horizon scalar pert. can be obtained generically for **any** MG theory.

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4 \chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Deep super-horizon volume and matter dynamics

- Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- Restrict to deep super-horizon modes but at **early times**.

single \longrightarrow Dynamic equations
spin for $\langle \hat{V} \rangle_{\sigma_{x^0}}, \langle \hat{\Phi} \rangle_{\sigma_{x^0}}$

Modified gravity

- Dynamics of deep super-horizon scalar pert. can be obtained generically for **any** MG theory.

Perturbing background dynamics

- Study deep super-horizon scalar pert. by perturbing background QG volume equation.

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Deep super-horizon volume and matter dynamics

- Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
 - Restrict to deep super-horizon modes but at **early times**.
- $\xrightarrow[\text{spin}]{\text{single}}$ Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}, \langle \hat{\Phi} \rangle_{\sigma_{x^0}}$

Modified gravity

- Dynamics of deep super-horizon scalar pert. can be obtained generically for **any** MG theory.

Perturbing background dynamics

- Study deep super-horizon scalar pert. by perturbing background QG volume equation.

No matching at early times with full effective GFT volume dynamics: **different d.o.f.!**

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar isotropic perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar isotropic perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

$$\begin{aligned} \delta V_\psi &\propto \text{Re}(\delta\Psi, \tilde{\sigma}\tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2) \\ \delta\phi_\psi &= \bar{\phi}_\psi (\delta V_\psi / \bar{V}_\psi) \end{aligned}$$

- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in $\delta\Phi$).

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra: better coupling of the physical frame. **Two-sector** (+, -) GFT (BC)!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar isotropic perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

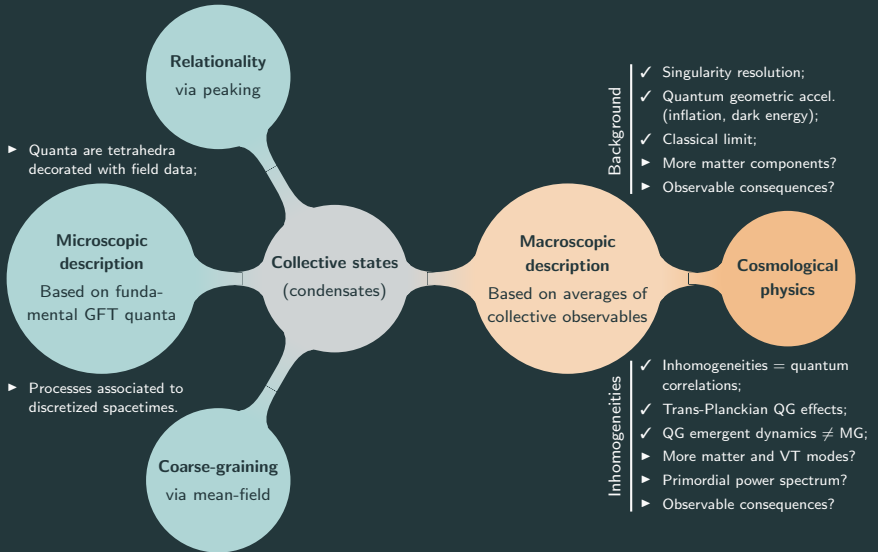
$$\delta V_\psi \propto \text{Re}(\delta\Psi, \tilde{\sigma}\tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2)$$

$$\delta\phi_\psi = \bar{\phi}_\psi (\delta V_\psi / \bar{V}_\psi)$$

- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in $\delta\Phi$).

Late times scalar isotropic perturbations

- ▶ QG corrections at (relationally) trans-Planckian scales.
- ▶ GR matching at larger scales.



Backup

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or, in some cases, $G = \mathrm{SU}(2)$.

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

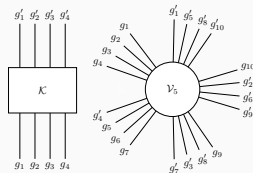
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{V_{\gamma}}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{V_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \nu_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

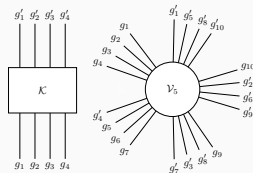
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{V_{\gamma}}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{V_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \nu_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

- Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

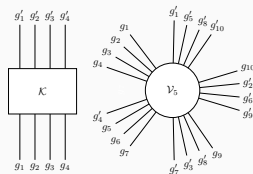
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{complete spinfoam model.}$$

- Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V}_{γ} chosen to match the desired spinfoam model.

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

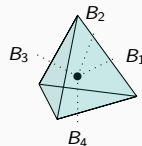
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

$$\blacktriangleright X \cdot B_a = 0 \text{ (BC).}$$



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

$$\sum_a B_a = 0$$

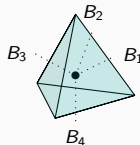
(faces of the tetrahedron close).

Simplicity

THIS TALK

$$\triangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

$$\triangleright X \cdot B_a = 0 \text{ (BC).}$$



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Lie group (connection)

$$\mathcal{H}_{\Delta_a} = L^2(G)$$

Non-comm.

FT

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

THIS TALK

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

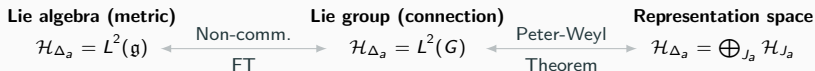
$$\blacktriangleright X \cdot B_a = 0 \text{ (BC).}$$



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

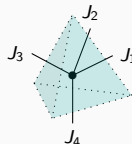
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
- ▶ $X \cdot B_a = 0$ (BC).

THIS TALK

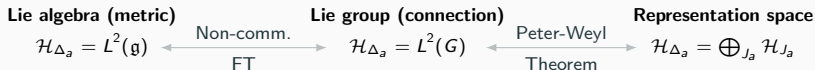


Group Field Theory and Loop Quantum Gravity

Fundamental quanta

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

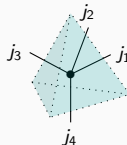
Closure

$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- THIS TALK
- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
 - ▶ $X \cdot B_a = 0$ (BC).



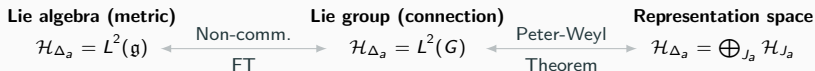
LQG

- ▶ Impose simplicity and reduce to $G = \text{SU}(2)$.
- ▶ Impose closure (gauge invariance).

Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

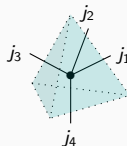
Closure

$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- THIS TALK
- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
 - ▶ $X \cdot B_a = 0$ (BC).



- ▶ Impose simplicity and reduce to $G = \text{SU}(2)$.
- ▶ Impose closure (gauge invariance).

$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv} \left[\bigotimes_{a=1}^4 \mathcal{H}_{j_a} \right]$$

= open spin-network vertex space

The Group Field Theory Fock space

Tetrahedron wavefunction


$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

The Group Field Theory Fock space

Tetrahedron wavefunction

$\varphi(g_1, \dots, g_4)$
(subject to constraints)

Many-body
Theory 

GFT field operator

$\hat{\varphi}(g_1, \dots, g_4)$
(subject to constraints)

The Group Field Theory Fock space

Tetrahedron wavefunction

$\varphi(g_1, \dots, g_4)$
(subject to constraints)

Many-body
Theory \longrightarrow

GFT field operator

$\hat{\varphi}(g_1, \dots, g_4)$
(subject to constraints)

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- ▶ Similar to \mathcal{H}_{LQG} but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

GFT Fock space

The Group Field Theory Fock space

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

Many-body
Theory

GFT field operator

$$\hat{\varphi}(g_1, \dots, g_4)$$

(subject to constraints)

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- ▶ Similar to \mathcal{H}_{LQG} but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

GFT Fock space

Operators

Volume operator $\hat{V} = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \ell} V_{j_a, \ell} \hat{\varphi}_{j_a, m_a, \ell}^\dagger \hat{\varphi}_{j_a, m_a, \ell}.$

- ▶ Generic second quantization prescription to build a $m + n$ -body operator: sandwich matrix elements between spin-network states between m powers of $\hat{\varphi}^\dagger$ and n powers of $\hat{\varphi}$.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi}) \quad \hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi}) \quad \hat{H}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{\chi^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
functionals of $\tilde{\sigma}$
localized at x^0 .

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\vec{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
functionals of $\tilde{\sigma}$
localized at x^0 .

Relationality

► Averaged evolution wrt x^0 is physical:

$$\text{Intensive} \longleftarrow \langle \hat{X}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{H}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
functionals of $\tilde{\sigma}$
localized at x^0 .

Wavefunction
→
isotropy

$$\langle \hat{V} \rangle_{\sigma_x^0} = \sum_v V_v |\tilde{\sigma}_v|^2(x^0)$$

$$\langle \hat{H} \rangle_{\sigma_x^0} = \sum_v |\tilde{\sigma}_v|^2(x^0)$$

► $v = j \in \mathbb{N}/2$ (EPRL);
► $v = \rho \in \mathbb{R}$ (ext. BC).

Relationality

► Averaged evolution wrt x^0 is physical:

$$\langle \hat{X}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{H}^0 \rangle_{\sigma_{x^0}}$.

The main ingredients

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \, \chi \int dg_a \, \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle .$$

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_l, x^\alpha)} \right\rangle_\sigma = \int d h_a \int d \chi \mathcal{K}(g_a, h_a, (x^\alpha - \chi^\alpha)^2) \sigma(h_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_l, x^\alpha)} \right\rangle_\sigma = \int d h_a \int d \chi \mathcal{K}(g_a, h_a, (x^\alpha - \chi^\alpha)^2) \sigma(h_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

Relationality

Condensate Peaked States

- ▶ Constructing relational observables in full QG is difficult (QFT with no continuum intuition).

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \, \chi \int d g_a \, \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_l, x^\alpha)} \right\rangle_\sigma = \int d h_a \int d \chi \, \mathcal{K}(g_a, h_a, (x^\alpha - \chi^\alpha)^2) \sigma(h_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

Relationality

Condensate Peaked States

- ▶ Constructing relational observables in full QG is difficult (QFT with no continuum intuition).
- ▶ Relational localization implemented at an **effective** level on observable **averages** on condensates.
- ▶ If χ^μ constitute a physical reference frame, this can be achieved by assuming
$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$$

Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ $\mathcal{D} = \text{minisuperspace} + \text{clock}$.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ $\mathcal{D} = \text{minisuperspace} + \text{clock}$.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \oint_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V} \right)^2 \simeq \left(\frac{2 \oint_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \oint_v V_v \rho_v^2} \right)^2, \quad \frac{V''}{V} \simeq \frac{2 \oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ \mathcal{D} = minisuperspace + clock.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \oint_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V} \right)^2 \simeq \left(\frac{2 \oint_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \oint_v V_v \rho_v^2} \right)^2, \quad \frac{V''}{V} \simeq \frac{2 \oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Classical limit (large N , late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ \mathcal{D} = minisuperspace + clock.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \oint_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V} \right)^2 \simeq \left(\frac{2 \oint_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \oint_v V_v \rho_v^2} \right)^2, \quad \frac{V''}{V} \simeq \frac{2 \oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Classical limit (large N , late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

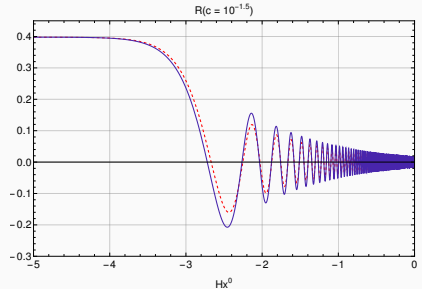
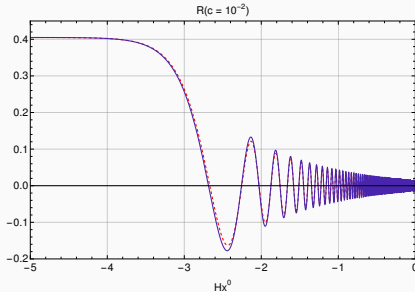
- ▶ Quantum fluctuations on clock and geometric variables are under control.

Bounce

- ▶ A non-zero volume bounce happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

Dynamics of a comoving curvature-like variable

$$\tilde{\mathcal{R}} \equiv -\frac{\delta V}{3\bar{V}} + \mathcal{H} \frac{\delta\phi}{\bar{\phi}'}, \quad \tilde{\mathcal{R}}'' - a^4 \nabla^2 \tilde{\mathcal{R}} = - \left[\mathcal{H} - \frac{1}{4} (\bar{\phi}^2)' \right] \left(\frac{\delta V}{\bar{V}} \right)'.$$



- c = ratio between volume and matter pert. initial conditions.

- Deep sub-horizon modes are trans-Planckian in the physical reference frame.

As c increases, QG dynamics (blue line) differs more and more from the GR one (red dotted line) at (relational) trans-Planckian scales.