

Scalar cosmological perturbations from full quantum gravity

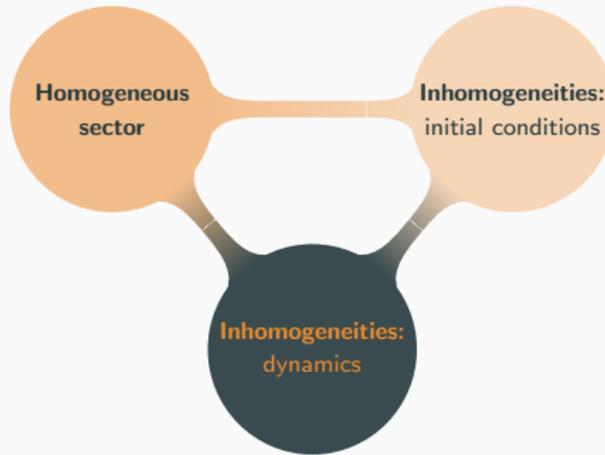
In collaboration with: D. Oriti, E. Wilson-Ewing, A. Pithis, A. Jercher, P. Fischer

Luca Marchetti

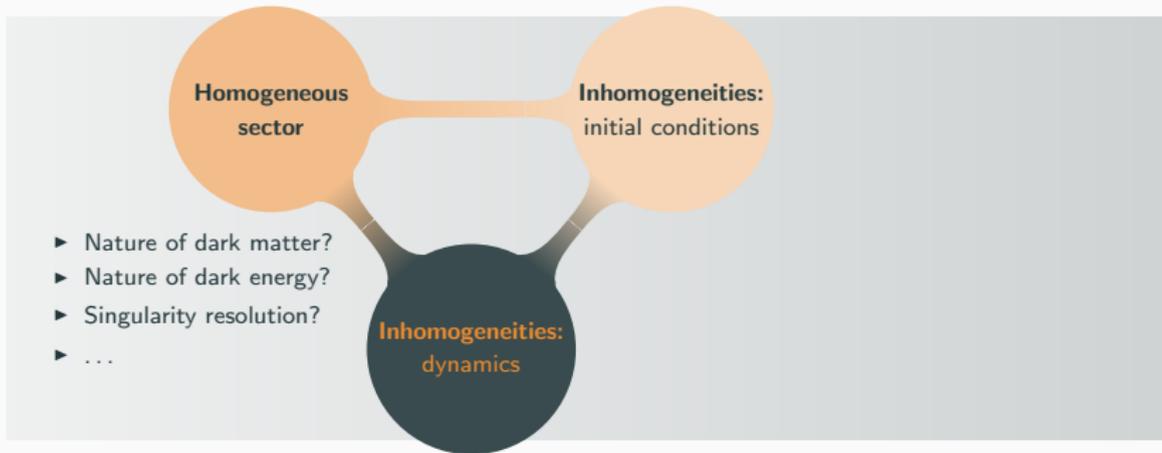
Theory Canada 15
Mount Allison University
17 June 2023

Department of Mathematics and Statistics
UNB Fredericton

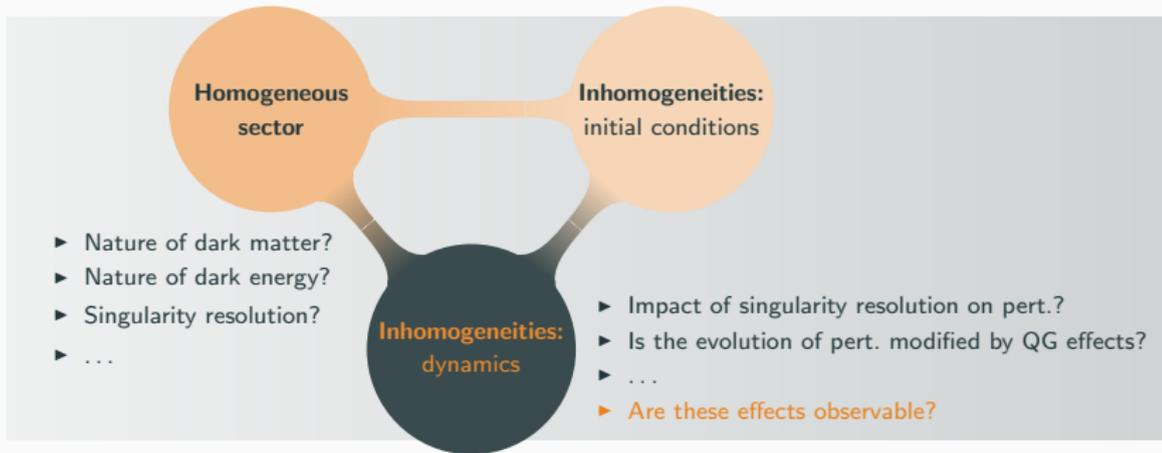
The QG perspective on Cosmology



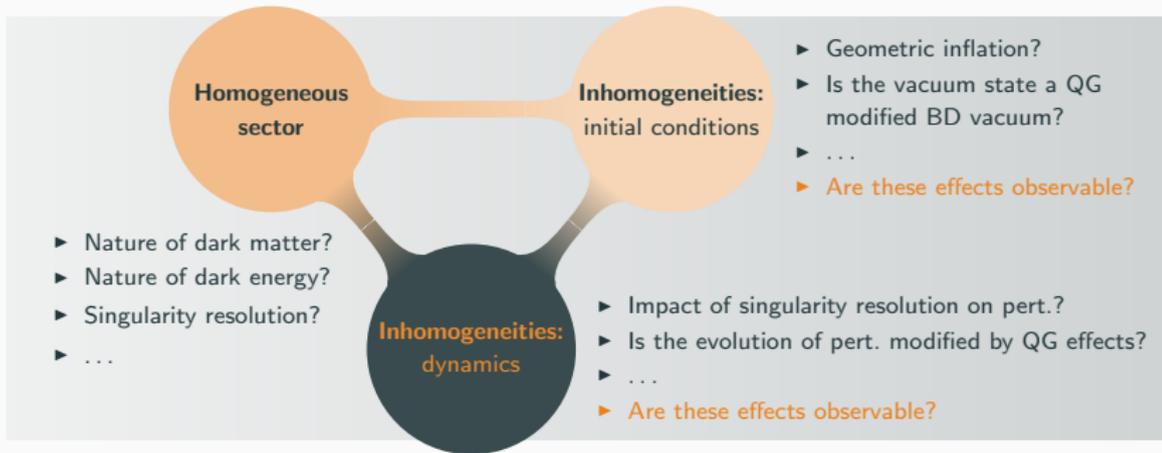
The QG perspective on Cosmology



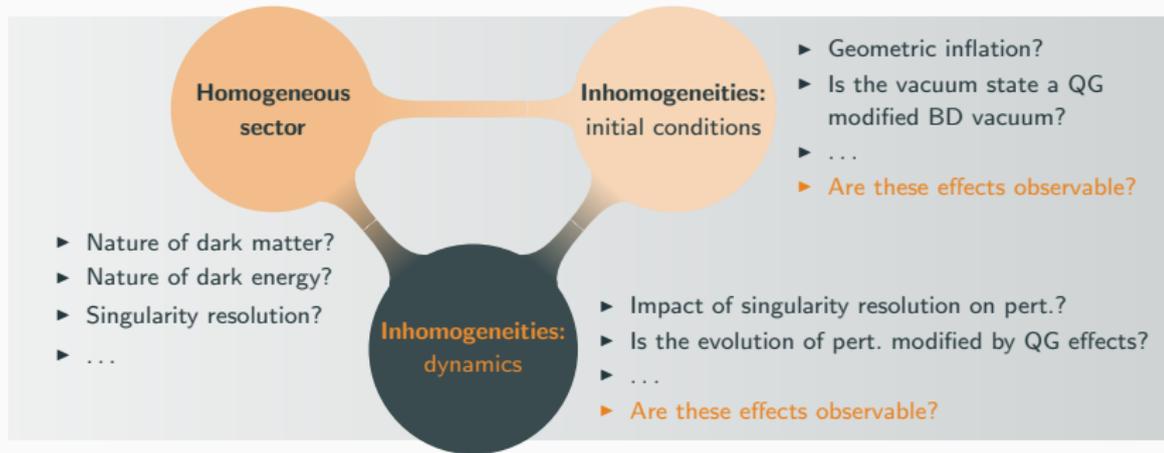
The QG perspective on Cosmology



The QG perspective on Cosmology



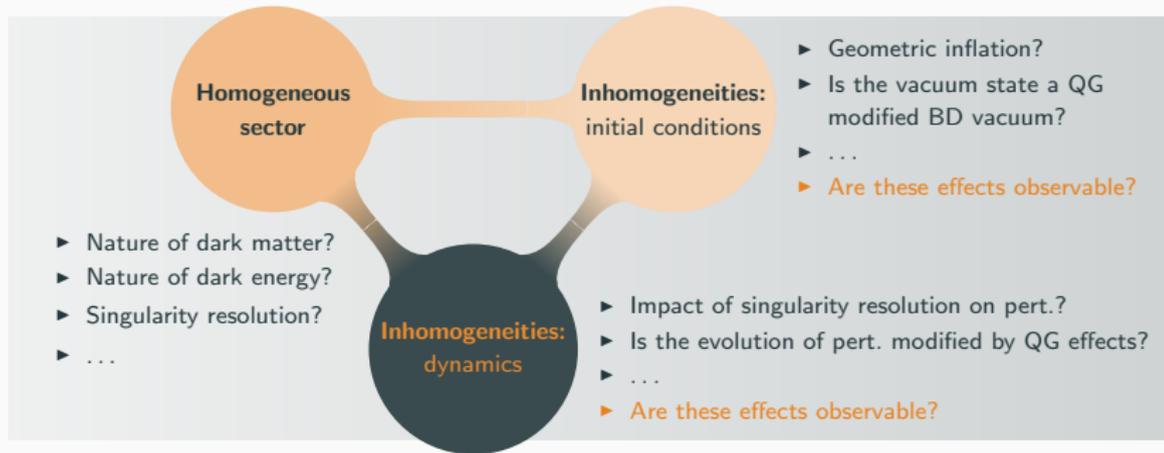
The QG perspective on Cosmology



Challenges in background independent and emergent QG:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
- ▶ ...

The QG perspective on Cosmology

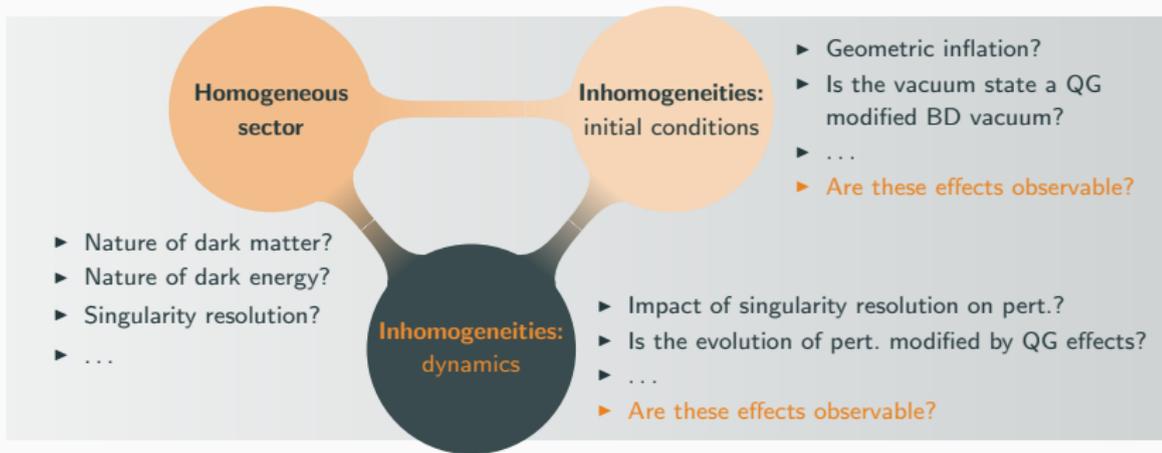


Challenges in background independent and emergent QG:

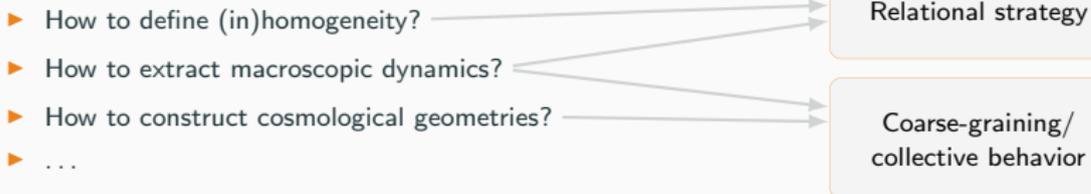
- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
- ▶ ...

Relational strategy

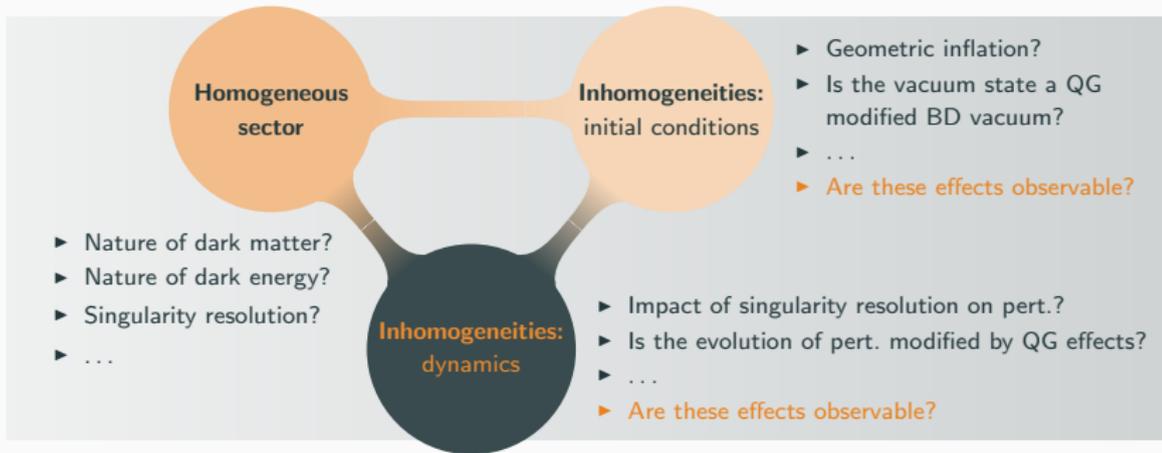
The QG perspective on Cosmology



Challenges in background independent and emergent QG:



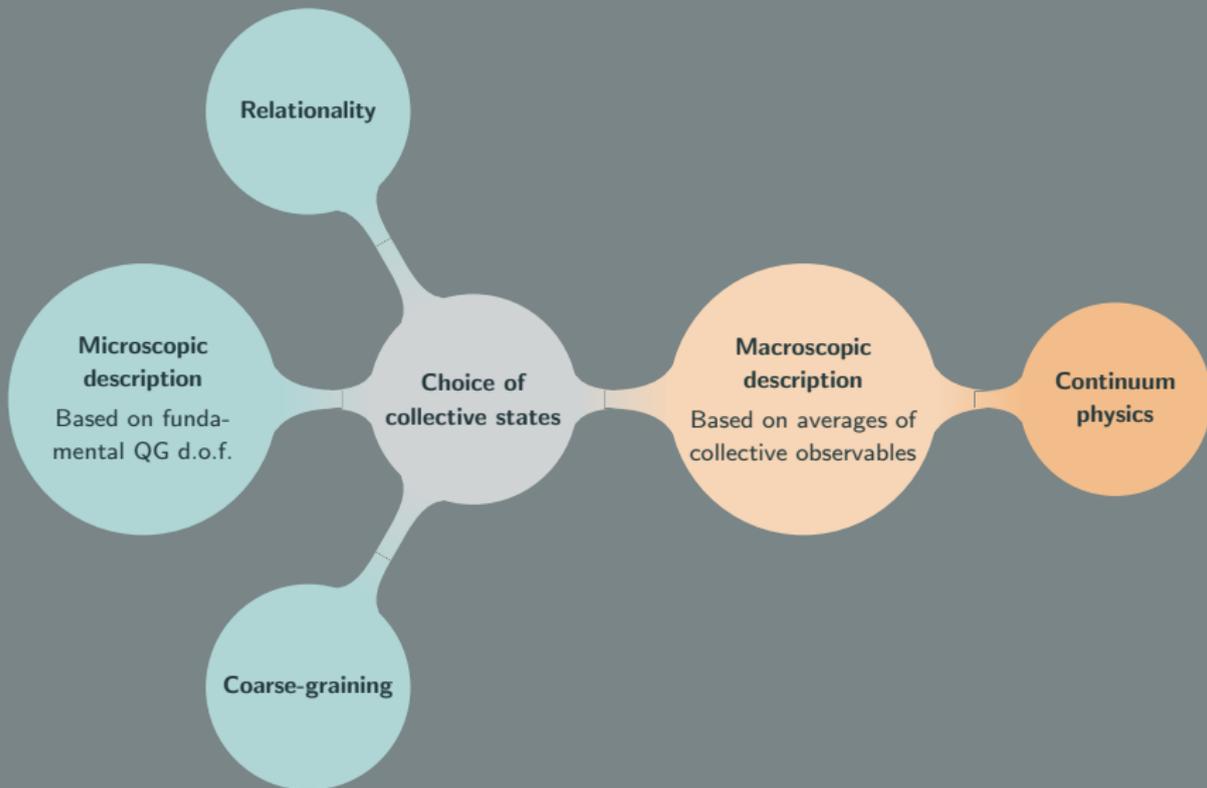
The QG perspective on Cosmology

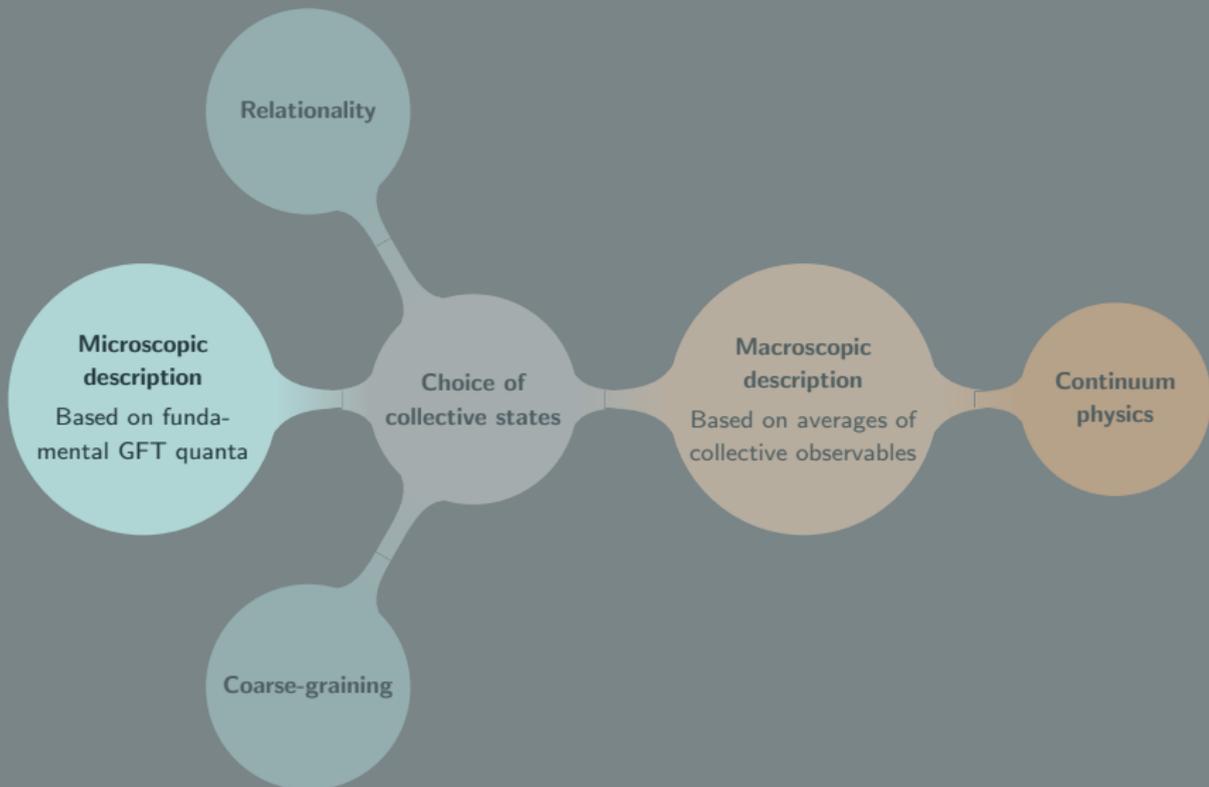


Challenges in background independent and emergent QG:

- ▶ How to define (in)homogeneity? → Relational strategy
- ▶ How to extract macroscopic dynamics? → Relational strategy
- ▶ How to extract macroscopic dynamics? → Coarse-graining/collective behavior
- ▶ How to construct cosmological geometries? → Coarse-graining/collective behavior
- ▶ ...

Approximate only





Introduction to Group Field Theory

The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the "spacetime to be" ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

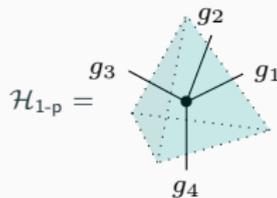
The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:



The (T)GFT approach to quantum gravity

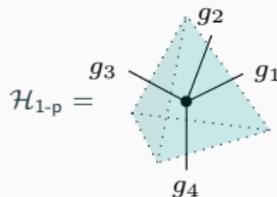
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.



The (T)GFT approach to quantum gravity

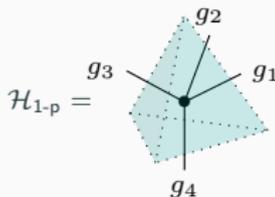
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.



The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:

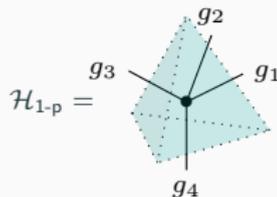
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.



Processes: Discrete spacetimes

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

The (T)GFT approach to quantum gravity

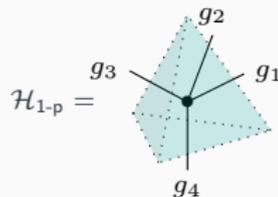
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.



Processes: Discrete spacetimes

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

- ▶ **Non-local and combinatorial** interactions mimic the gluing of $d - 1$ -simplices into d -simplices.
- ▶ Γ are **dual to spacetime triangulations**.

$$Z_{\text{GFT}} = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{discrete gravity path-integral.}$$



The (T)GFT approach to quantum gravity

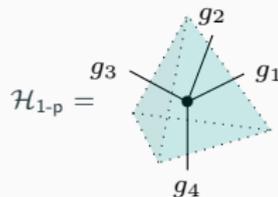
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.



Processes: Discrete spacetimes

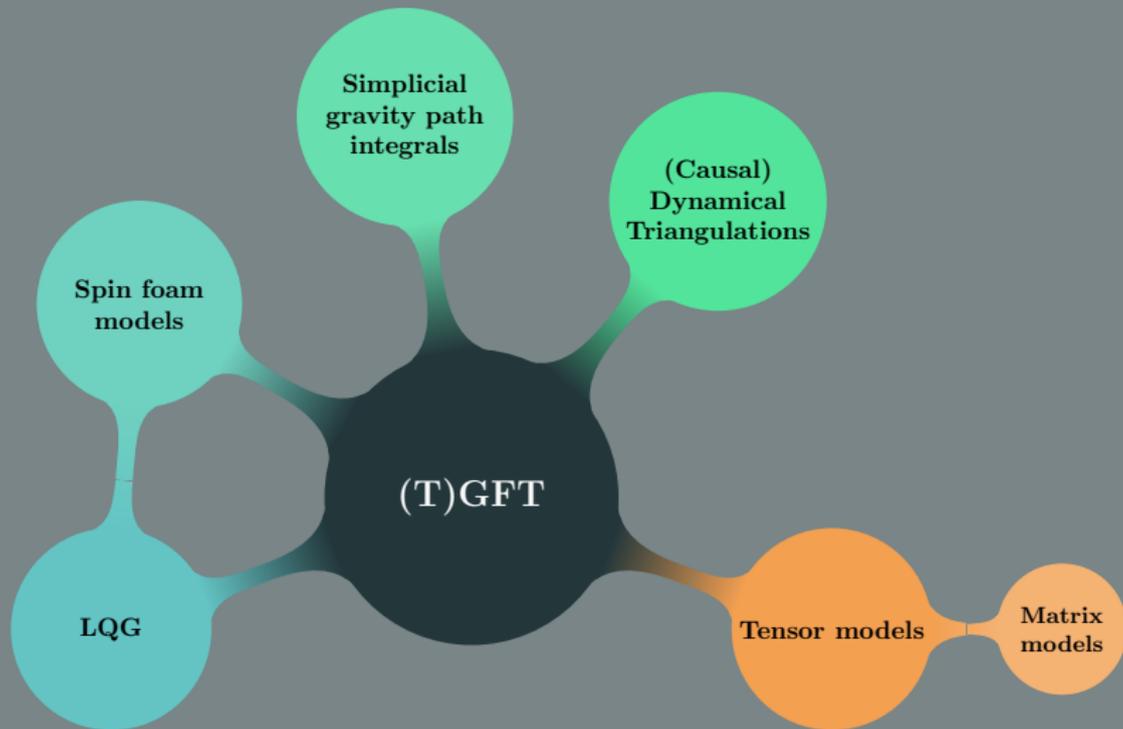
S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

- ▶ **Non-local and combinatorial** interactions mimic the gluing of $d - 1$ -simplices into d -simplices.
- ▶ Γ are **dual to spacetime triangulations**.

$$Z_{\text{GFT}} = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{discrete gravity path-integral.}$$



GFTs are QFTs of atoms of spacetime.



Group Field Theory and matter: scalar fields

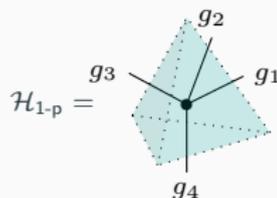
Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on the product G^d .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ **Geometricity constraints** imposed analogously as before.



Processes: Discrete spacetimes

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- ▶ Geometric data enter interactions in a **non-local and combinatorial** fashion.

Group Field Theory and matter: scalar fields

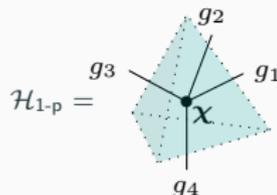
Group Field Theories: theories of a field $\varphi : G^d \times \mathbb{R}^d \rightarrow \mathbb{C}$ defined on the product of G^d and \mathbb{R}^d .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ **Geometricity constraints** imposed analogously as before.
- ▶ Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



Processes: Discrete spacetimes

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- ▶ Geometric data enter interactions in a **non-local and combinatorial** fashion.
- ▶ Scalar field data are **local** in interactions.
- ▶ For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi^\alpha, \chi^{\alpha'}) = \mathcal{K}(g_a, g_b; (\chi^\alpha - \chi^{\alpha'})^2)$$
$$\mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)}, \chi) = \mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)})$$

Group Field Theory and matter: scalar fields

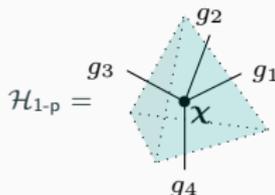
Group Field Theories: theories of a field $\varphi : G^d \times \mathbb{R}^d \rightarrow \mathbb{C}$ defined on the product of G^d and \mathbb{R}^d .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some models, $G = \text{SU}(2)$.

Quanta: Spacetime atoms

Quanta are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ **Geometricity constraints** imposed analogously as before.
- ▶ Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



Processes: Discrete spacetimes

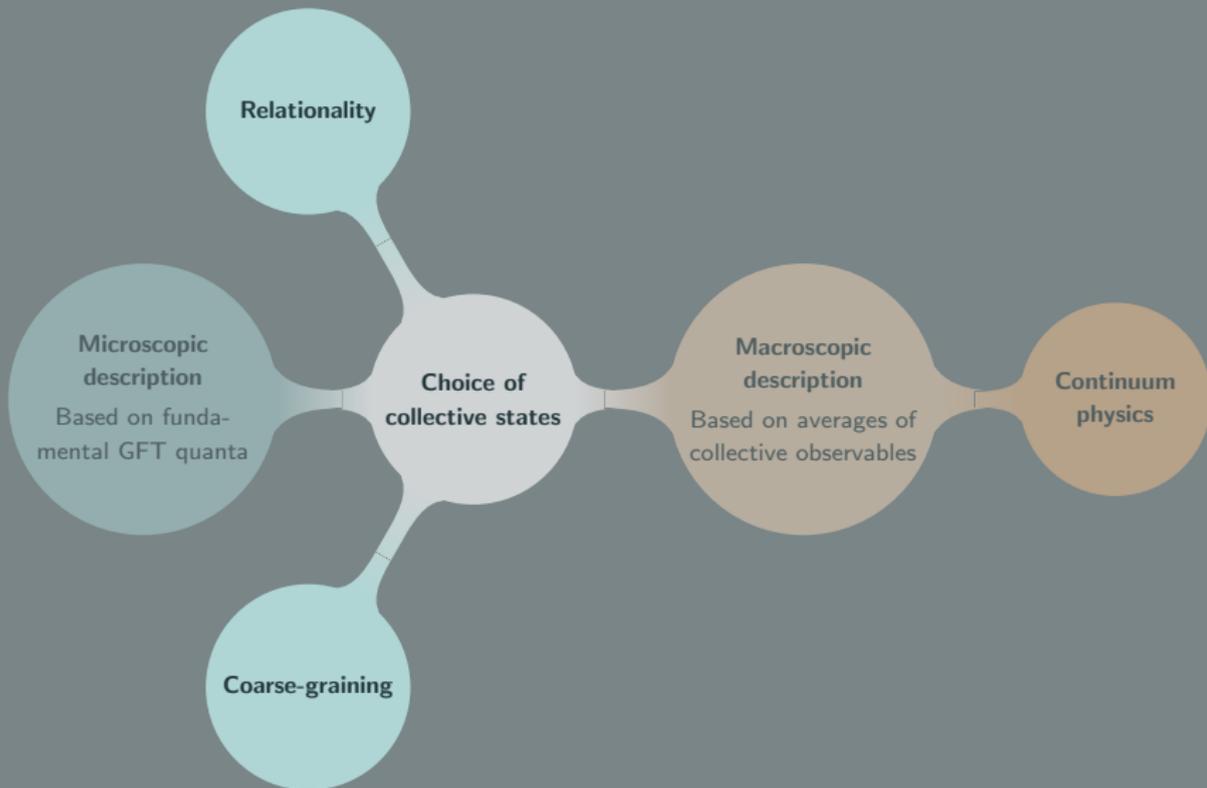
S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- ▶ Geometric data enter interactions in a **non-local and combinatorial** fashion.
- ▶ Scalar field data are **local** in interactions.
- ▶ For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi^\alpha, \chi^{\alpha'}) = \mathcal{K}(g_a, g_b; (\chi^\alpha - \chi^{\alpha'})^2)$$

$$\mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)}, \chi) = \mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)})$$

Domain of GFTs is the space of (discretized) continuum fields



The main ingredients

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\varphi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle.$$

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\varphi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_l, \chi^\alpha)} \right\rangle_\sigma = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (\chi^\alpha - \chi^\alpha)^2) \sigma(\mathbf{h}_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, \chi^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\varphi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_l, \chi^\alpha)} \right\rangle_\sigma = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (\chi^\alpha - \chi^\alpha)^2) \sigma(\mathbf{h}_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, \chi^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

Relationality

Condensate Peaked States

- ▶ Constructing relational observables in full QG is difficult (QFT with no continuum intuition).

The main ingredients

Collective states

GFT condensates

- ▶ From the GFT perspective, continuum geometries are associated to large number of quanta.
- ▶ The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\varphi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle.$$

Effective dynamics

Mean-field approximation

- ▶ When interactions are small (certainly satisfied in an appropriate regime) the dynamics of σ is:

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_l, x^\alpha)} \right\rangle_\sigma = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (x^\alpha - \chi^\alpha)^2) \sigma(\mathbf{h}_a, \chi^\alpha) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, x^\alpha)} \Big|_{\varphi=\sigma} = 0.$$

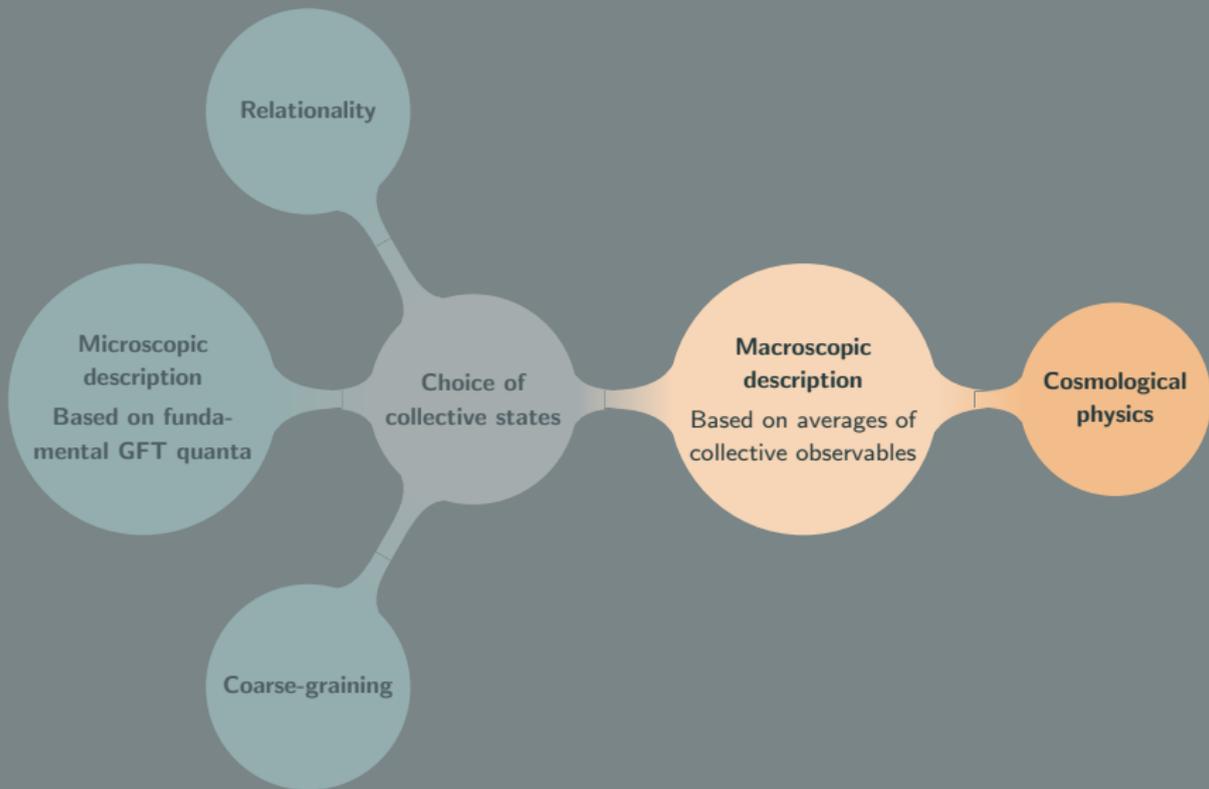
- ▶ Equivalent to **mean-field** (saddle-point) approx. of Z_{GFT} (reliable for physical models).

Relationality

Condensate Peaked States

- ▶ Constructing relational observables in full QG is difficult (QFT with no continuum intuition).
- ▶ Relational localization implemented at an **effective** level on observable **averages** on condensates.
- ▶ If χ^μ constitute a physical reference frame, this can be achieved by assuming

$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$$



Group Field Theory Cosmology

Homogeneous sector



Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ $\mathcal{D} = \text{minisuperspace} + \text{clock}$.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_v^f V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational homogeneous volume dynamics

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ \mathcal{D} = minisuperspace + clock.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \int_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2 \int_v V_v \rho_v \operatorname{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \int_v V_v \rho_v^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2 \int_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\int_v V_v \rho_v^2}$$

Effective relational homogeneous volume dynamics

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ \mathcal{D} = minisuperspace + clock.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \int \! \! \int_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2 \int_v V_v \rho_v \operatorname{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \int_v V_v \rho_v^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2 \int_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\int_v V_v \rho_v^2}$$

Classical limit (large N , late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

Effective relational homogeneous volume dynamics

Assumptions

(Relational) Homogeneity

- ▶ σ depends on a single clock MCFM field χ^0 .
- ▶ \mathcal{D} = minisuperspace + clock.

Volume operator captures the relevant physics:

Isotropy

- ▶ σ depends only on a single rep. label v .
- ▶ $v \in \mathbb{N}/2$ (EPRL-like) or $v \in \mathbb{R}$ (ext. BC).

$$V \equiv \langle \hat{V} \rangle_{\sigma_{x^0}} = \int \! \! \! \int_v V_v \rho_v^2(x^0), \quad \rho \equiv |\tilde{\sigma}|.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2 \int_v V_v \rho_v \operatorname{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3 \int_v V_v \rho_v^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2 \int_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\int_v V_v \rho_v^2}$$

Classical limit (large N , late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

Bounce

- ▶ A **non-zero volume bounce** happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

Inhomogeneous sector (dynamics)

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Quantum

- ▶ $\varphi(\mathbf{g}_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} respecting the classical matter symmetries.

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Quantum

- ▶ $\varphi(\mathbf{g}_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} respecting the classical matter symmetries.

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\mathbf{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

Mat. Vol. Frame

Scalar perturbations from GFT condensates

Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCMF **reference** fields (χ^0, χ^i) ,
- ▶ 1 MCMF **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\mathbf{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

Mat. Vol. Frame

Quantum

- ▶ $\varphi(\mathbf{g}_a, \chi^\mu, \phi)$ depends on 5 discretized scalar variables and is associated to **spacelike** tetrahedra.
- ▶ S_{GFT} respecting the classical matter symmetries.

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d g_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
 - ▶ Decoupling for a range of values of CPSs and large N (late times).
- single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{x^0}}$.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d g_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- ▶ Decoupling for a range of values of CPSs and large N (late times).

single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{x^0}}$.

Background

- ✓ Matching with GR possible.
- ▶ Macro. couplings defined in terms of GFT ones.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d g_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):

$$\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu), \quad \theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu).$$

Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
 - ▶ Decoupling for a range of values of CPSs and large N (late times).
- single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}, \langle \hat{\Phi} \rangle_{\sigma_{x^0}}$.

Background

- ✓ Matching with GR possible.
- ▶ Macro. couplings defined in terms of GFT ones.

Perturbations

- ✓ Large scales ("super-horizon") GR matching.

Scalar perturbations from GFT condensates

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d g_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Late times volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
 - ▶ Decoupling for a range of values of CPSs and large N (late times).
- single label \longrightarrow Dynamic equations for $\langle \hat{V} \rangle_{\sigma_{x^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{x^0}}$.

Background

- ✓ Matching with GR possible.
- ▶ Macro. couplings defined in terms of GFT ones.

Perturbations

- ✓ Large scales ("super-horizon") GR matching.
- ▶ **Unphysical behavior** of spatial derivative terms.

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Super-horizon volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- ▶ Restrict to super-horizon modes but study also early times.

single \longrightarrow Dynamic equations
spin \longrightarrow for $\langle \hat{V} \rangle_{\sigma_{\chi^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{\chi^0}}$

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Super-horizon volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- ▶ Restrict to super-horizon modes but study also early times.

single \longrightarrow Dynamic equations
spin \longrightarrow for $\langle \hat{V} \rangle_{\sigma_{\chi^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{\chi^0}}$

Modified gravity

- ▶ Dynamics of super-horizon scalar perturbations can be obtained generically for **any** MG theory.
- ▶ **No matching** at early times with effective GFT volume dynamics.

Super-horizon scalar perturbations

Mat. Vol. Frame

Observables

notation: $(\cdot, \cdot) = \int d^4\chi d\phi d\bar{g}_a$

$$\hat{X}^\mu = (\hat{\varphi}^\dagger, \chi^\mu \hat{\varphi}) \quad \hat{\Pi}^\mu = -i(\hat{\varphi}^\dagger, \partial_\mu \hat{\varphi})$$

Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

$$\hat{\Phi} = (\hat{\varphi}^\dagger, \phi \hat{\varphi}) \quad \hat{\Pi}_\phi = -i(\hat{\varphi}^\dagger, \partial_\phi \hat{\varphi})$$

States

- ▶ CPSs around $\chi^\mu = x^\mu$, with
 - η : **Isotropic** peaking on rods;
 - $\tilde{\sigma}$: **Isotropic** distribution of geometric data.
- ▶ Small relational $\tilde{\sigma}$ -inhomogeneities ($\tilde{\sigma} = \rho e^{i\theta}$):
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$, $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$.

Super-horizon volume and matter dynamics

- ▶ Averaged q.e.o.m. (no interactions) \longrightarrow coupled eqs. for (ρ, θ) .
- ▶ Restrict to super-horizon modes but study also early times.

single \longrightarrow Dynamic equations
spin \longrightarrow for $\langle \hat{V} \rangle_{\sigma_{x^0}}$, $\langle \hat{\Phi} \rangle_{\sigma_{x^0}}$

Modified gravity

- ▶ Dynamics of super-horizon scalar perturbations can be obtained generically for **any** MG theory.
- ▶ **No matching** at early times with effective GFT volume dynamics.

Perturbing background dynamics

- ▶ Study super-horizon scalar perturbations by perturbing background QG volume equation.
- ▶ **No matching** at early times with full effective GFT volume dynamics.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \hat{\delta}\Phi \otimes \mathbb{I}_- + \hat{\delta}\Psi + \mathbb{I}_+ \otimes \hat{\delta}\Xi) |0\rangle$$

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

- ▶ Late times and single (spacelike) rep. label.

$$\delta V_\psi \propto \text{Re}(\delta\Psi, \tilde{\sigma} \tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2)$$

- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in $\delta\Phi$).

Scalar perturbations from quantum correlations

Two-body correlations

Including timelike tetrahedra allows to better couple the physical frame: **two-sector** (+, -) GFT!

$$|\psi\rangle = \mathcal{N}_\psi \exp(\hat{\sigma} \otimes \mathbb{I}_- + \mathbb{I}_+ \otimes \hat{\tau} + \widehat{\delta\Phi} \otimes \mathbb{I}_- + \widehat{\delta\Psi} + \mathbb{I}_+ \otimes \widehat{\delta\Xi}) |0\rangle$$

Background

- ▶ $\hat{\sigma} = (\sigma, \hat{\varphi}_+^\dagger)$: spacelike condensate.
- ▶ $\hat{\tau} = (\tau, \hat{\varphi}_-^\dagger)$: timelike condensate.
- ▶ τ, σ peaked; $\tilde{\tau}, \tilde{\sigma}$ homogeneous.

Perturbations

- ▶ $\widehat{\delta\Phi} = (\delta\Phi, \hat{\varphi}_+^\dagger \hat{\varphi}_+^\dagger)$, $\widehat{\delta\Psi} = (\delta\Psi, \hat{\varphi}_+^\dagger \hat{\varphi}_-^\dagger)$, $\widehat{\delta\Xi} = (\delta\Xi, \hat{\varphi}_-^\dagger \hat{\varphi}_-^\dagger)$.
- ▶ $\delta\Phi, \delta\Psi$ and $\delta\Xi$ small and relationally inhomogeneous.
- ▶ Perturbations = nearest neighbour 2-body **correlations**.

Scalar perturbations

- ▶ 2 mean-field eqs. for 3 variables ($\delta\Phi, \delta\Psi, \delta\Xi$):

$$\langle \delta S / \delta \hat{\varphi}_+^\dagger \rangle_\psi = 0 = \langle \delta S / \delta \hat{\varphi}_-^\dagger \rangle_\psi$$

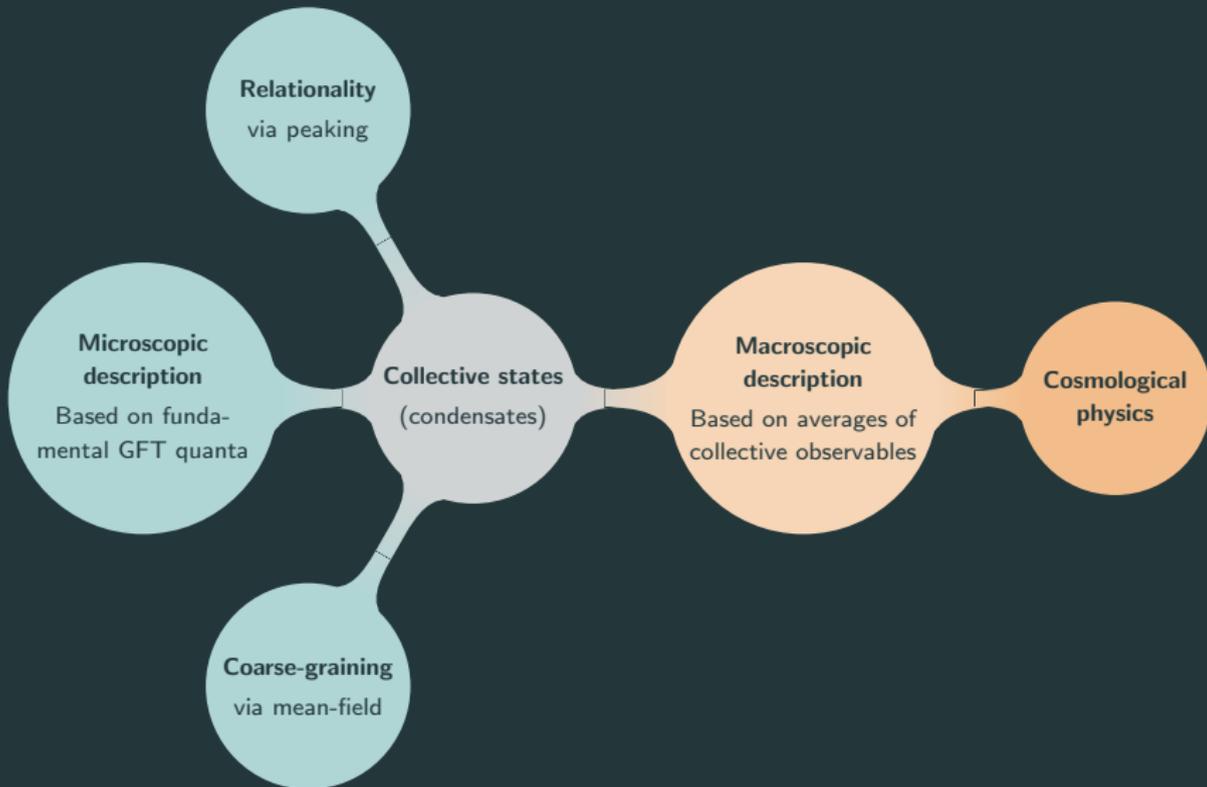
- ▶ Late times and single (spacelike) rep. label.

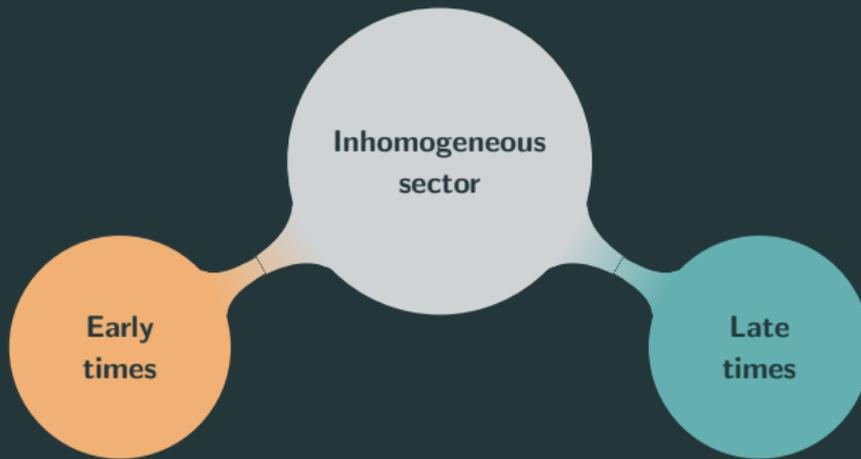
$$\delta V_\psi \propto \text{Re}(\delta\Psi, \tilde{\sigma} \tilde{\tau}) + \text{Re}(\delta\Phi, \tilde{\sigma}^2)$$

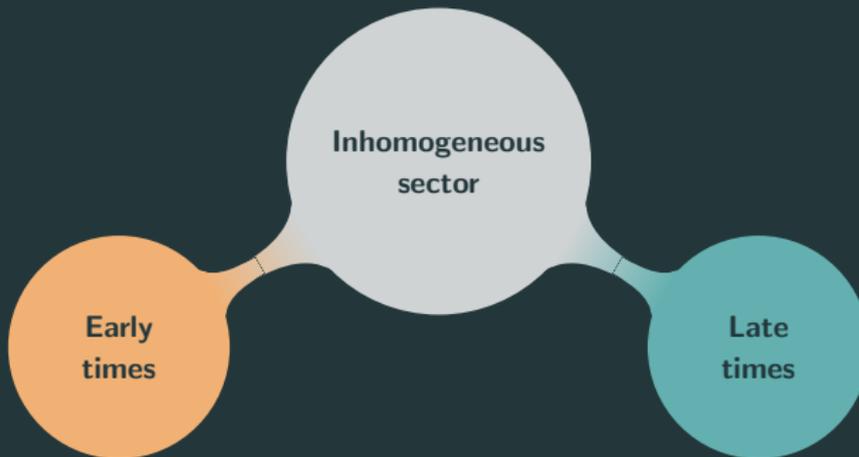
- ▶ Physical behavior of spatial derivative terms fixes dynamical freedom (e.g. in $\delta\Phi$).

Late times volume perturbations

- ▶ QG corrections to trans-Planckian modes dynamics.
- ▶ GR matching at larger scales.

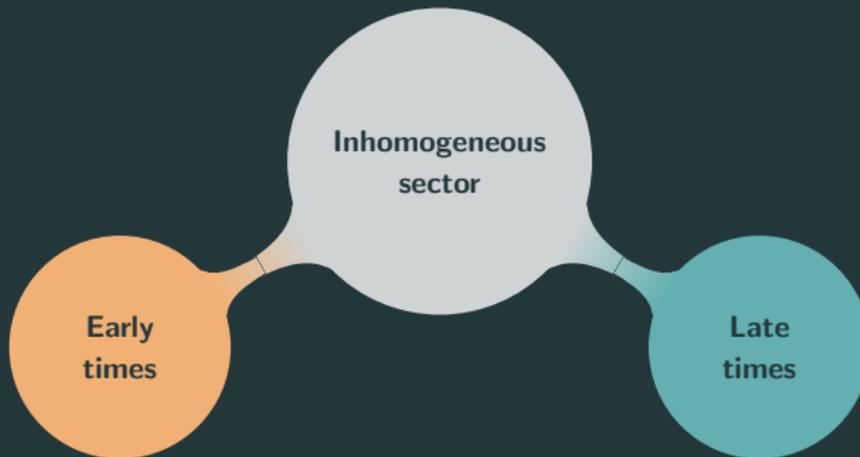






Results

- ✓ Super-horizon analysis with MCMF scalar fields:
 - ✓ Scalar pert. dynamics differs from any MG model.
 - ✓ Full QG scalar pert. dynamics differs from QG perturbed background one.

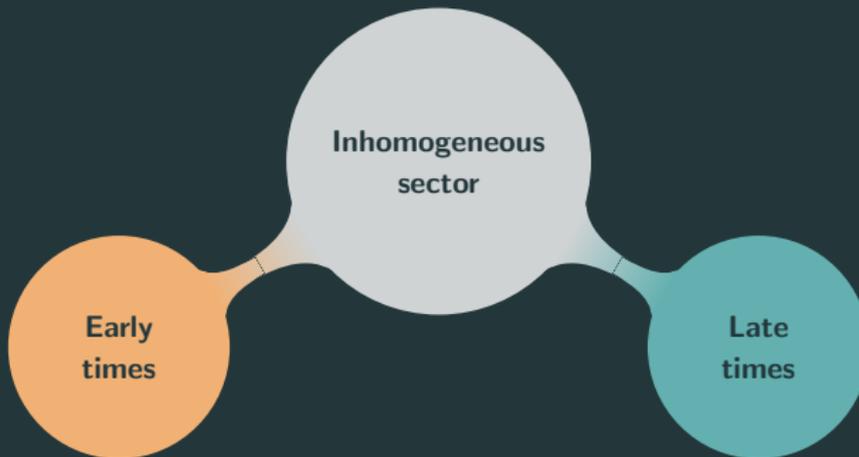


Results

- ✓ Super-horizon analysis with MCMF scalar fields:
 - ✓ Scalar pert. dynamics differs from any MG model.
 - ✓ Full QG scalar pert. dynamics differs from QG perturbed background one.

Results

- ✓ All scales analysis with MCMF scalar fields:
 - ✓ Manifest causal properties of quanta allow for a careful coupling of the physical ref. frame.
 - ✓ Scalar pert. \longleftrightarrow quantum correlations!
 - ✓ Late-times volume pert. dynamics matches GR at large scales. . .
 - ✓ . . . but receives corrections for trans-Planckian modes!



Perspectives

- ⚠ Different fundamental d.o.f. \rightarrow different perturbation dynamics?
- ⚠ Scalar field perturbations? EFT description?
 - ▶ Generalization to physically interesting fluids.
 - ▶ Extension to VT modes: more observables!
 - ▶ Initial conditions and power spectra?
 - Fock quantization of early-times dynamics.
 - Can we derive it from full QG?

Perspectives

- ⚠ Physical (perhaps observable) consequences of trans-Planckian mismatch?
- ⚠ Scalar field perturbations? EFT description?
 - ▶ Generalization to physically interesting fluids.
 - ▶ Extension to VT modes: more observables!
 - ▶ How do quantum perturbations classicalize?

Backup

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

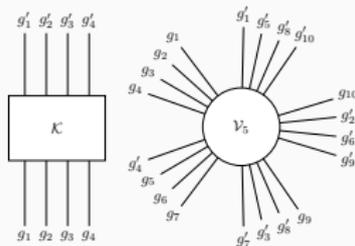
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.}$$

- ▶ Interaction terms are **combinatorially non-local**.
- ▶ Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}).$$



Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

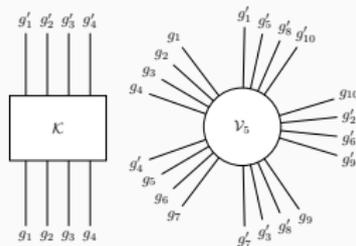
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.}$$

- ▶ Interaction terms are **combinatorially non-local**.
- ▶ Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}).$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

- ▶ Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- ▶ Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.

Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined on d copies of a group manifold G .

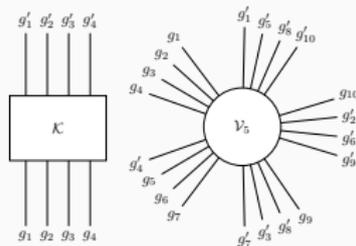
d is the dimension of the “spacetime to be” ($d = 4$) and G is the local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or, in some cases, $G = \text{SU}(2)$.

Action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] + \text{c.c.}$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\mathcal{V}_{\gamma}}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}_{\gamma}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}).$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{complete spinfoam model.}$$

- Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V}_{γ} chosen to match the desired spinfoam model.

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

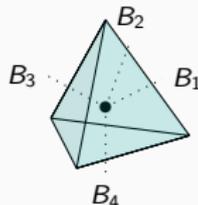
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

$$\blacktriangleright X \cdot B_a = 0 \text{ (BC).}$$



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

$$\sum_a B_a = 0$$

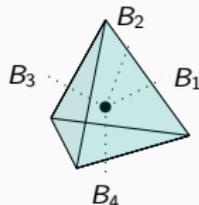
(faces of the tetrahedron close).

Simplicity

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

$$\blacktriangleright X \cdot B_a = 0 \text{ (BC).}$$

THIS TALK



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Lie algebra (metric)

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$$

Non-comm.

FT

Lie group (connection)

$$\mathcal{H}_{\Delta_a} = L^2(G)$$

Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

$$\sum_a B_a = 0$$

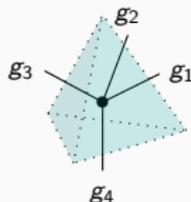
(faces of the tetrahedron close).

Simplicity

$$\blacktriangleright X \cdot (B - \gamma \star B)_a = 0 \text{ (EPRL);}$$

$$\blacktriangleright X \cdot B_a = 0 \text{ (BC).}$$

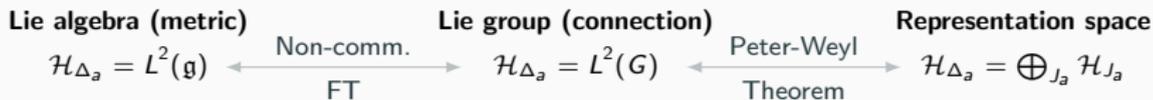
THIS TALK



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

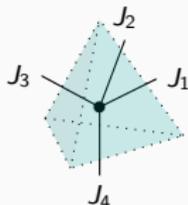
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
- ▶ $X \cdot B_a = 0$ (BC).

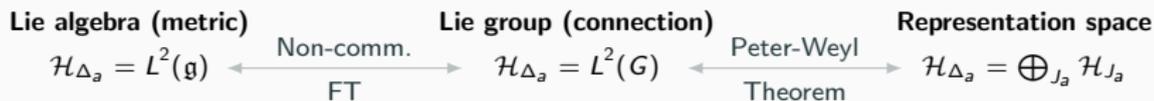
THIS TALK



Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

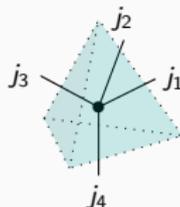
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
- ▶ $X \cdot B_a = 0$ (BC).

THIS TALK

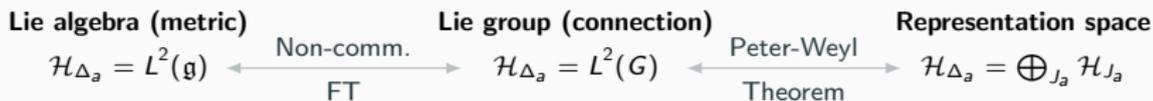


- ▶ Impose simplicity and reduce to $G = \text{SU}(2)$.
- ▶ Impose closure (gauge invariance).

Group Field Theory and Loop Quantum Gravity

One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \otimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)



Constraints

Geometricity constraints (appropriately encoded in \mathcal{K} and \mathcal{V}_γ) allow for a $d - 1$ -simplicial interpretation of the fundamental quanta:

Closure

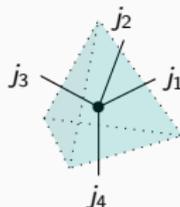
$$\sum_a B_a = 0$$

(faces of the tetrahedron close).

Simplicity

- ▶ $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
- ▶ $X \cdot B_a = 0$ (BC).

THIS TALK



- ▶ Impose simplicity and reduce to $G = \text{SU}(2)$.
- ▶ Impose closure (gauge invariance).

$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv} \left[\bigotimes_{a=1}^4 \mathcal{H}_{j_a} \right]$$

= open spin-network vertex space

The Group Field Theory Fock space

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

The Group Field Theory Fock space

Tetrahedron wavefunction

$\varphi(g_1, \dots, g_4)$
(subject to constraints)

Many-body
Theory \rightarrow

GFT field operator

$\hat{\varphi}(g_1, \dots, g_4)$
(subject to constraints)

The Group Field Theory Fock space

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

Many-body
Theory

GFT field operator

$$\hat{\varphi}(g_1, \dots, g_4)$$

(subject to constraints)

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

GFT Fock space

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- ▶ Similar to \mathcal{H}_{LQG} but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

The Group Field Theory Fock space

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

Many-body
Theory \longrightarrow

GFT field operator

$$\hat{\varphi}(g_1, \dots, g_4)$$

(subject to constraints)

GFT Fock space

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.
- ▶ Similar to \mathcal{H}_{LQG} but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

Operators

Volume operator $\hat{V} = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \ell} V_{j_a, \ell} \hat{\varphi}_{j_a, m_a, \ell}^\dagger \hat{\varphi}_{j_a, m_a, \ell}$

- ▶ Generic second quantization prescription to build a $m + n$ -body operator: sandwich matrix elements between spin-network states between m powers of $\hat{\varphi}^\dagger$ and n powers of $\hat{\varphi}$.

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0

(\mathcal{D} = minisuperspace + clock)

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi}) \quad \hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi}) \quad \hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

Spatial relational homogeneity:

σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
functionals of $\tilde{\sigma}$
localized at x^0 .

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:
 σ depends on a MCMF “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
functionals of $\tilde{\sigma}$
localized at x^0 .

Relationality

► Averaged evolution wrt x^0 is physical:

$$\text{Intensive} \longleftarrow \langle \hat{X}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:
 σ depends on a MCMF “clock” scalar field χ^0
 (\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$:
 functionals of $\tilde{\sigma}$
 localized at x^0 .

Wavefunction
 $\xrightarrow{\text{isotropy}}$

$$\langle \hat{V} \rangle_{\sigma_{x^0}} = \sum_v V_v |\tilde{\sigma}_v|^2(x^0)$$

$$\langle \hat{N} \rangle_{\sigma_{x^0}} = \sum_v |\tilde{\sigma}_v|^2(x^0)$$

► $v = j \in \mathbb{N}/2$ (EPRL);

► $v = \rho \in \mathbb{R}$ (ext. BC).

Relationality

► Averaged evolution wrt x^0 is physical:

$$\langle \hat{X}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}}$.