# Relational physics from an emergent quantum gravity perspective

In collaboration with: D. Oriti, E. Wilson-Ewing, S. Gielen, A. Polaczek

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# Overview

- **The relational strategy**
  - The classical and quantum perspectives
  - The emergent perspective and effective methods
- **Introduction to Group Field Theory**
  - Connection with spinfoam models and LQG
  - Including scalar matter
  - Relational strategy and GFTs
- **Effective approaches**
  - General considerations
  - Coherent Peaked States
  - State-agnostic approach
- **Conclusions**

# The relational strategy

# Relational strategy: the classical and quantum GR perspective

**Background independence** $\longleftrightarrow$ **Problem of localization** $\longleftrightarrow$ **Relational strategy**

Quite well understood from a classical perspective, less from a quantum perspective.

Isham 9210011; Rovelli Class. Quantum Grav. 8 297; Dittrich 0507106; Tambornino 1109.0740; Giesel, Thiemann 0711.0119 . . .

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### Classical

Physical localization via relational observables:

- Take two phase space functions, $f$ and $T$ with $\{T, C_H\} \neq 0$ ($T$ relational clock).
- The relational extension $F_{f,T}(\tau)$ of $f$ encodes the value of $f$ when $T$ reads $\tau$.
- Evolution in $\tau$ is relational.
- $F_{f,T}(\tau)$ is a very complicated function.
- Applications almost only for very simple systems.

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### Quantum GR

Dirac approach: Quantize first.

- Perspective neutral.
- Poor control of the physical Hilbert space.

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**Quantum GR**

Dirac approach: Quantize first.

- Perspective neutral.
- Poor control of the physical Hilbert space.

Reduced approach: Relationality first.

- No quantum constraint to solve.
- Not perspective neutral. Too complicated to implement in most of the cases.

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A genuinely new dimension of the problem arises for emergent QG theories.

LM, Oriti 2008.02774; Giulini 0603087; Kuchar Int.J.Mod.Phys.D 20(2011); Isham 9210011; Rovelli Class. Quantum Grav. 8 297;

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- Fundamental d.o.f. are weakly related to spacetime quantities;
- The latter expected to emerge from the former in an appropriate phase.

**Macroscopic proto-geo**

- Set of collective observables;
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**Effective approaches:**

- More mathematical control and physical insights.
- Relevant for observative purposes.

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# Introduction to Group Field Theory

# Group Field Theory and spinfoam models

Definition

Group Field Theories: theories of a field $\varphi : G^d \to \mathbb{C}$ defined on $d$ copies of a group manifold $G$.

$d$ is the dimension of the "spacetime to be" ($d = 4$) and $G$ is the local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or, in some cases, $G = \mathrm{SU}(2)$.

Oriti 1110.5606; Reisenberger, Rovelli 0002083; De Pietri, Petronio 0004045; Gurau 1006.0714; Baratin, Oriti 1002.4723; Finocchiaro, Oriti 1812.03550.

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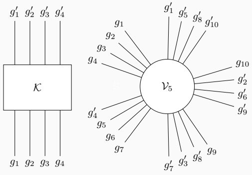
Action

$$S[\varphi,\bar{\varphi}] = \int \mathrm{d}g_a \bar{\varphi}(g_a)\mathcal{K}[\varphi](g_a) + \sum_\gamma \frac{\lambda_\gamma}{n_\gamma} \mathrm{Tr}_{\mathcal{V}_\gamma}[\varphi] + \text{c.c.}\,.$$

- Interaction terms are combinatorially non-local.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph $\gamma$:

$$\mathrm{Tr}_{\mathcal{V}_\gamma}[\varphi] = \int \prod_{i=1}^{n_\gamma} \mathrm{d}g_a \prod_{(a,i;b,j)} \mathcal{V}_\gamma(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_\gamma} \varphi(g_a^{(i)})\,.$$

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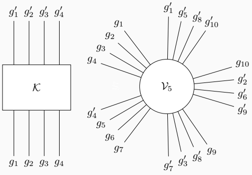


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## Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_\Gamma w_\Gamma(\{\lambda_\gamma\}) A_\Gamma$$

- $\Gamma$ = stranded diagrams dual to $d$-dimensional cellular complexes of arbitrary topology.
- Amplitudes $A_\Gamma$ = sums over group theoretic data associated to the cellular complex.

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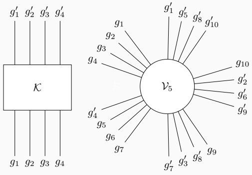


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$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_\Gamma(\{\lambda_\gamma\}) A_\Gamma = \text{ complete spinfoam model.}$$

- $\Gamma$ = stranded diagrams dual to $d$-dimensional cellular complexes of arbitrary topology.
- Amplitudes $A_\Gamma$ = sums over group theoretic data associated to the cellular complex.
- $\mathcal{K}$ and $\mathcal{V}_\gamma$ chosen to match the desired spinfoam model.


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Fundamental quanta

## One-particle Hilbert space

The one-particle Hilbert space is $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^{4} \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints)

Finocchiaro, Oriti 1812.03550; Baez, Barrett 9903060; Baratin, Oriti 1002.4723; Gielen, Oriti 1004.5371; Oriti 1310.7786.

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**Lie algebra (metric)**

$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{g})$

### Constraints

Geometricity constraints (appropriately encoded in $\mathcal{K}$ and $\mathcal{V}_\gamma$) allow for a $d-1$-simplicial interpretation of the fundamental quanta:

**Closure**

$\sum_a B_a = 0$
(faces of the tetrahedron close).

**Simplicity**

- $X \cdot (B - \gamma \star B)_a = 0$ (EPRL);
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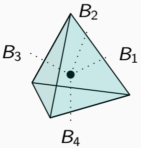


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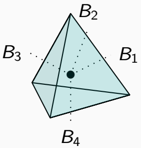


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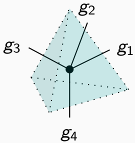


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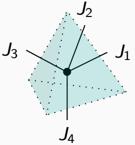

# Group Field Theory and Loop Quantum Gravity

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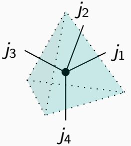


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- Impose simplicity and reduce to $G = \mathrm{SU}(2)$.
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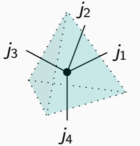


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$$\mathcal{H}_{\text{tetra}} = \bigoplus_{\vec{j}} \text{Inv}\left[\bigotimes_{a=1}^{4} \mathcal{H}_{j_a}\right]$$
$$= \text{open spin-network vertex space}$$

Finocchiaro, Oriti 1812.03550; Baez, Barrett 9903060; Baratin, Oriti 1002.4723; Gielen, Oriti 1004.5371; Oriti 1310.7786.

# The Group Field Theory Fock space

**Tetrahedron wavefunction**

$\varphi(g_1, \ldots, g_4)$
(subject to constraints)

Oriti 1310.7786; Oriti 1408.7112; Sahlman, Sherif 2302.03612.

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$\xrightarrow[\text{Theory}]{\text{Many-body}}$

**GFT field operator**

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Many-body $\longrightarrow$ Theory

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GFT Fock space

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym}\left[\mathcal{H}^{(1)}_{\text{tetra}} \otimes \mathcal{H}^{(2)}_{\text{tetra}} \otimes \ldots \mathcal{H}^{(V)}_{\text{tetra}}\right]$$

- $\mathcal{F}_{\text{GFT}}$ generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, $\mathcal{H}_\Gamma$ space of states associated to connected simplicial complexes $\Gamma$.
- Generic states do not correspond to connected simplicial lattices nor classical simplicial geometries.
- Similar to $\mathcal{H}_{\text{LQG}}$ but also different: no continuum intuition, orthogonality wrt nodes, not graphs.

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**Operators**

Volume operator $\hat{V} = \int \mathrm{d}g_a^{(1)}\, \mathrm{d}g_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{j_a, m_a, \iota} V_{j_a, \iota} \hat{\varphi}^\dagger_{j_a, m_a, \iota} \hat{\varphi}_{j_a, m_a, \iota}$.

- Generic second quantization prescription to build a $m+n$-body operator: sandwich matrix elements between spin-network states between $m$ powers of $\hat{\varphi}^\dagger$ and $n$ powers of $\hat{\varphi}$.

Oriti 1310.7786; Oriti 1408.7112; Sahlman, Sherif 2302.03612.

# Group Field Theory and matter: scalar fields

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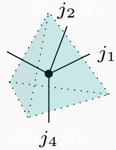
### Kinematics

Quanta are $d - 1$-simplices decorated with quantum geometric and scalar data:

- Geometricity constraints imposed analogously as before.

$\mathcal{H}_{\text{tetra}} =$ 


### Dynamics

$S_{\text{GFT}}$ obtained by comparing $Z_{\text{GFT}}$ with simplicial gravity + scalar fields path integral.

- Geometric data enter the action in a non-local and combinatorial fashion.

Group Field Theories: theories of a field $\varphi : G^d \times \mathbb{R}^{d_l} \to \mathbb{C}$ defined on the product of $G^d$ and $\mathbb{R}^{d_l}$.

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## Kinematics

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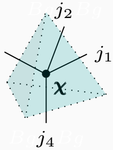
- Geometricity constraints imposed analogously as before.
- Scalar field discretized on each $d$-simplex: each $d - 1$-simplex composing it carries values $\boldsymbol{\chi} \in \mathbb{R}^{d_l}$.

$\mathcal{H}_{\text{tetra}} =$

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- Geometric data enter the action in a non-local and combinatorial fashion.
- Scalar field data are local in interactions.
- For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi^\alpha, \chi^{\alpha\prime}) = \mathcal{K}(g_a, g_b; (\chi^\alpha - \chi^{\alpha\prime})^2)$$

$$\mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)}, \boldsymbol{\chi}) = \mathcal{V}_5(g_a^{(1)}, \dots, g_a^{(5)})$$

Li, Oriti, Zhang 1701.08719; Oriti 0912.2441; Gielen, Oriti 1311.1238; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Sindoni 1602.08104; ...

# Relational strategy and the GFT Fock space

# Relational strategy in GFT: difficulties

Ante quantum

**Classical**

- $N$ classical GFT atoms: $\mathcal{C}^{(i)} = G^d \times \mathbb{R}^{d_l}$.
- $i$th-atom deparametrizable wrt. a clock $\chi^{0,(i)}$.
- Synchronize the clocks $\chi^{0,(i)} \longrightarrow t_N$.
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Post quantum

### Relational observables?

How to construct them without having manifest access to diffeos?

$\downarrow$

Simplest ansatz: localize operators wrt. clock data.

$$\hat{N} = \int \mathrm{d}g_a \, \mathrm{d}\chi \, \hat{\varphi}^\dagger(g_a, \chi) \hat{\varphi}(g_a, \chi) \,,$$

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**Open questions**

A scalar field should be represented as an operator on $\mathcal{F}_{\text{GFT}}$.

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$\chi =$ eigenvalue on "synchronous" states.

- What about "non-synchronous" states?
- Extension to generic observables?

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The pre-geometric, many-body nature of GFTs hinders the implementation of the relational strategy!

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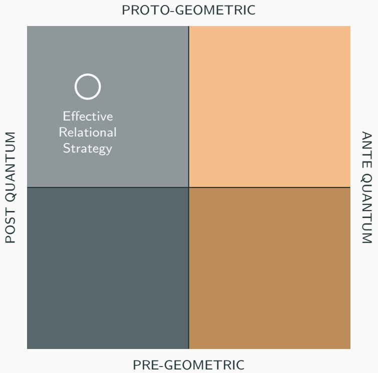
# Effective approaches

# Emergent effective relational strategy

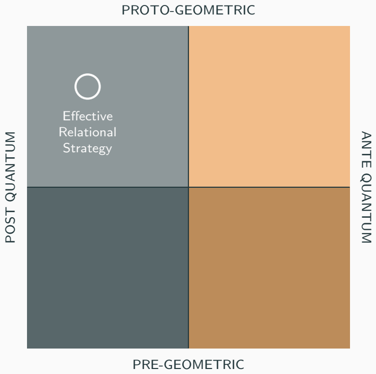



**Basic principles**

**Emergence** Relational strategy in terms of collective observables and states.

**Effectiveness** Averaged relational localization. Internal frame not too quantum.

LM, Oriti 2008.02774; Bojowald, Hoehn, Tsobanjan 1011.3040; Bojowald, Tsobanjan 0906.1772;

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**Concrete example: scalar field clock**

## Emergence

- Identify (collective) states $|\Psi\rangle$ admitting a continuum proto-geometric interpretation.
- Identify a set of collective observables:

$$\hat{O}_a \xrightarrow{\langle\cdot\rangle_\Psi} \text{Geometric observables}$$

$$\hat{\chi} \quad \hat{\Pi} \xrightarrow{\langle\cdot\rangle_\Psi} \text{Scalar field and its momentum}$$

$$\hat{N} \xrightarrow{\langle\cdot\rangle_\Psi} \text{Number of quanta}$$

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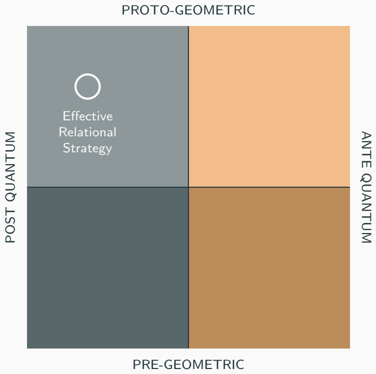


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## Effectivness

- It exists a "Hamiltonian" $\hat{H}$ such that

$$i\frac{\mathrm{d}}{\mathrm{d}\,\langle\hat{\chi}\rangle_\Psi}\langle\hat{O}_a\rangle_\Psi = \langle[\hat{H},\hat{O}_a]\rangle_\Psi\,,$$

and whose moments coincide with those of $\hat{\Pi}$.
- Relative fluctuations of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$:

$$\Delta^2\chi \ll 1\,, \qquad \Delta^2\chi \sim \langle\hat{N}\rangle_\Psi^{-1}\,.$$

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# Coherent Peaked States

# The main ingredients

Collective states

### GFT condensates

- From the GFT perspective, continuum geometries are associated to large number of quanta.
- The simplest states that can accommodate infinite number of quanta are condensate states:

$$|\sigma\rangle = \mathcal{N}_\sigma \exp\left[\int \mathrm{d}^{d_l}\chi \int \mathrm{d}g_a\, \sigma(g_a, \chi^\alpha)\hat{\varphi}^\dagger(g_a, \chi^\alpha)\right]|0\rangle\,.$$

LM, Oriti 2008.02774; LM, Oriti 2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238; Gielen 1404.2944.

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Effective dynamics

### Mean-field approximation

- When interactions are small (certainly satisfied in an appropriate regime) the dynamics of $\sigma$ is:

$$\left\langle \frac{\delta S[\hat{\varphi},\hat{\varphi}^\dagger]}{\delta\hat{\varphi}(g_I,x^\alpha)}\right\rangle_\sigma = \int \mathrm{d}h_a \int \mathrm{d}\chi\, \mathcal{K}(g_a,h_a,(x^\alpha-\chi^\alpha)^2)\sigma(h_a,\chi^\alpha) + \lambda \frac{\delta V[\varphi,\varphi^*]}{\delta\varphi^*(g_a,x^\alpha)}\bigg|_{\varphi=\sigma} = 0\,.$$

- Non-perturbative: equivalent to a mean-field (saddle-point) approximation of $Z$.

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### Condensate Peaked States

- Constructing relational observables on $\mathcal{F}_{\mathrm{GFT}}$ is difficult (QFT with no continuum intuition).

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Relationality

## Condensate Peaked States

- Constructing relational observables on $\mathcal{F}_{\text{GFT}}$ is difficult (QFT with no continuum intuition).
- Relational localization implemented at an effective level on observable averages.
- If $\chi^\mu$ constitute a reference frame, this can be achieved by assuming

$$\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$$

LM, Oriti 2008.02774; LM, Oriti 2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238; Gielen 1404.2944.

# Macroscopic cosmological variables and effective relationality

**Spatial relational homogeneity:**
$\sigma$ depends on a MCMF "clock" scalar field $\chi^0$

LM, Oriti 2008.02774 ; LM, Oriti 2010.09700.

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**Relationality**

## Clock expectation values

For large $N$, $x^0$ has a clear physical meaning:

$$\langle \hat{\chi}^0 \rangle_{\sigma_{x^0}} \equiv \langle \hat{X}^0 \rangle_{\sigma_{x^0}} / N \qquad \textit{(intensive)}$$

$$= x^0 \left(1 + \delta X(x^0)/N(x^0)\right)$$

$$\langle \hat{\Pi}^0 \rangle_{\sigma_{x^0}} = \langle \hat{H}_\sigma \rangle_{\sigma_{x^0}} \left(1 + \text{const.}/N(x^0)\right)$$

## Clock variances

For large $N$, clock fluctuations scale as $N^{-1}$:

$$\Delta^2_{\sigma_{x^0}} \chi^0 < \frac{1}{N}\left(1 + \frac{\epsilon}{2(x^0)^2} \frac{1}{(1+\delta X/N)^2}\right)$$

$$\Delta^2_{\sigma_{x^0}} \Pi^0 = \Delta^2_{\sigma_{x^0}} H_\sigma \left(1 + \text{const.}/N(x^0)\right)$$

$$\Delta^2_{\sigma_{x^0}} H_\sigma = \Delta^2_{\sigma_{x^0}} N = N^{-1}(x^0)\,.$$

LM, Oriti 2008.02774 ; LM, Oriti 2010.09700.

# Classical limit and validity of the framework

Effective dynamics

### Mean-field approximation

- Mesoscopic regime: large $N$ but negligible interactions.
- Derivative expansion of $\mathcal{K}$ (due to peaking properties).
- Isotropy: $\tilde{\sigma}_j \equiv \rho_j e^{i\theta_j}$ fundamental variables.

$$\tilde{\sigma}_j'' - 2i\tilde{\pi}_0\tilde{\sigma}_j' - E_j^2\tilde{\sigma} = 0.$$

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$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\sum_j V_j\rho_j \mathrm{sgn}(\rho_j')\sqrt{\mathcal{E}_j - Q_j^2/\rho_j^2 + \mu_j^2\rho_j^2}}{3\sum_j V_j\rho_j^2}\right)^2, \quad \frac{V''}{V} \simeq \frac{2\sum_j V_j\left[\mathcal{E}_j + 2\mu_j^2\rho_j^2\right]}{\sum_j V_j\rho_j^2}$$

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### Large number of quanta (large volume and late times)

- ✓ Volume quantum fluctuations under control.
- If $\mu_j^2$ is mildly dependent on $j$ (or one $j$ is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

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- ✓ $x^0 = \langle\hat{\chi}^0\rangle_{\sigma_{x^0}}$.
- ✓ Clock quantum fluctuations negligible.
- ✓ $\langle\hat{\Pi}^0\rangle_{\sigma_{x^0}} = \langle\hat{H}_\sigma\rangle_{\sigma_{x^0}}$ (higher moments $\simeq 0$).

Effective relational framework reliable!

LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881.

# State-agnostic approach

# Effective approach for constrained quantum systems

How does our scheme for extraction of relational cosmological physics depend on the specific choice of states? $\longleftrightarrow$ **A "state-agnostic" strategy is needed!**

LM, Gielen, Oriti, Polaczek 2110.11176; Bojowald, Sandhoefer, Skirzewski, Tsobanjan 0804.3365; Bojowald Tsobanjan 0906.1772.

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How does our scheme for extraction of relational cosmological physics depend on the specific choice of states? $\longleftrightarrow$ **A "state-agnostic" strategy is needed!**

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**Step 3: relational rewriting**

- Determine the remaining gauge flow which preserves the gauge conditions.
- Write evolution of the remaining variables wrt. $T$ (classical clock).

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# A state agnostic approach: application to GFT

How can this framework be generalized to a **field theory context**?

Infinitely many algebra generators.

Infinitely many quantum constraints.

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**Motivations**

- Interest in a coarse grained system characterized by a small number of macroscopic (1-body) observables.
- Expected to be the case for cosmology.

**Coarse-graining truncation**

- When the e.o.m. are linear, consider an integrated 1-body quantum constraint.
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GFT with MCMF scalar field

- Free e.o.m.: $\mathcal{D}\varphi \equiv (m^2 + \hbar^2\Delta_g + \lambda\hbar^2\,\partial_\chi^2)\varphi = 0$.
- Quantum constr. $\hat{C} = \int \hat{\varphi}^\dagger \mathcal{D}\hat{\varphi} = m^2\hat{N} - \hat{\Lambda} - \lambda\hat{\Pi}_2$.
- Generators: $\hat{X}$, $\hat{\Pi}$, $\hat{\Pi}_2$, $\hat{N}$, $\hat{\Lambda}$ and $\hat{K}$.
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Results

## Expectation values and variances

- The procedure can naturally be carried over by choosing as clock variable $\hat{K}$.
- Relational evolution of $\langle\hat{X}\rangle$ in agreement with classical cosmology.
- Fluctuations are decoupled from expect. values.
- If they are small at small $\langle\hat{K}\rangle$ they stay small even at large $\langle\hat{K}\rangle$ (probably associated to a constant $\langle\hat{N}\rangle$).

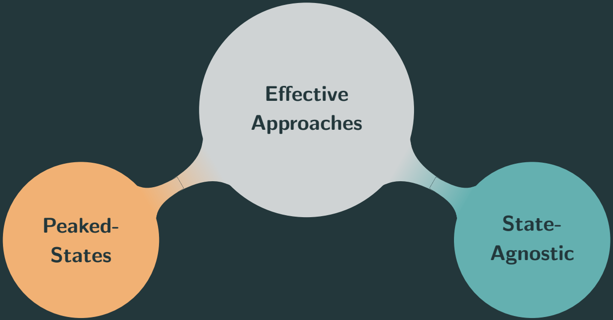
LM, Gielen, Oriti, Polaczek 2110.11176.

# Conclusions


Effective Approaches
Peaked-States
State-Agnostic


| **Results** | **Results** |
|---|---|
| ✓ Definition of an effective framework to implement the relational strategy: | ✓ Definition of an effective framework to implement the relational strategy: |
| ✓ Achieved via "synchronized" collective states. | ✓ Achieved via truncated constraints. |
| ✓ Naturally reliable in the classical limit. | ✓ Quantum effects small at low curvature. |
| ► Breaks down when quantum effects are large. | ► Quantum effects also small at high curvature! |
| ✓ Crucial role of the number operator identified. | ✓ Crucial role of the number operator identified. |

LM, Oriti 2008.02774; LM, Oriti 2010.09700; 2110.11176.




### Perspectives

- Extension to different physical reference frames.
  - Technically non-trivial.
  - Geometry peaking?
  - More geometry/matter observables needed!
- Comparison with state-agnostic approach.

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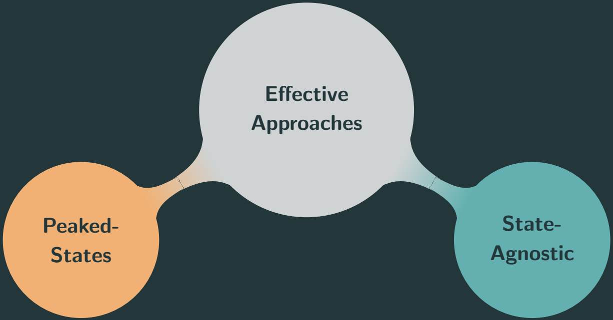


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⚠ Generalization of state-agnostic approach to QFTs (using covariant phase space and $n$PI techniques)!

De Vuyst, Höhn, LM, Mele (in progress).


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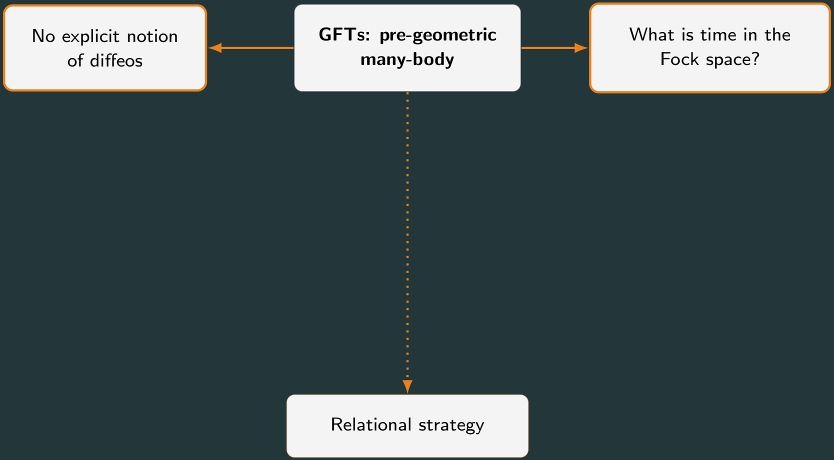
What about relational observables in full GFT?

De Vuyst, Höhn, LM, Mele (in progress).

**GFTs: pre-geometric many-body**

No explicit notion of diffeos ← **GFTs: pre-geometric many-body** → What is time in the Fock space?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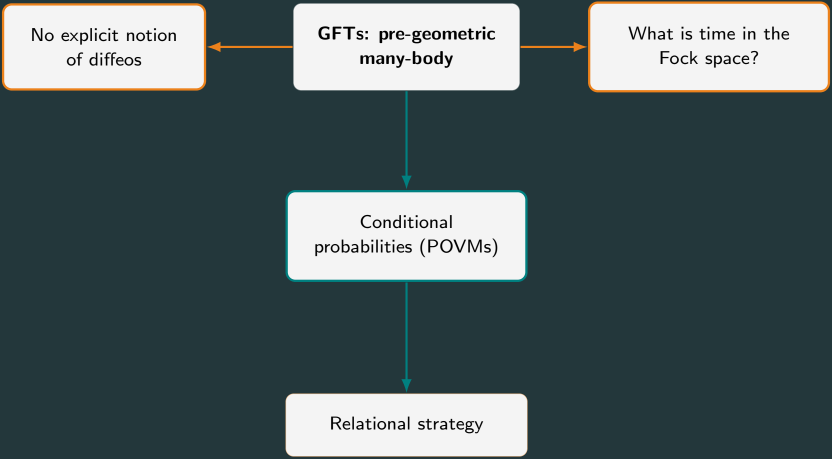
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Relational strategy

## Clock POVMs

There cannot exist a self-adjoint (monotonic) $\hat{T}$ canonically conjugate to a bounded $\hat{H}_C$.

LM, Oriti, Wilson-Ewing (in progress).

# Relational strategy in full GFT

Quantum Mechanics

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Group Field Theory

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$$\hat{E}_\chi = |0\rangle\langle 0| + \mathrm{d}\chi \sum_{n=1}^{\infty} \frac{1}{n!} \int \left[\prod_{i=1}^{n} \mathrm{d}\chi_i \, \mathrm{d}\xi_i\right] \frac{\sum_{i=1}^{n} \delta(\chi_i - \chi)}{n} \left[\prod_{i=1}^{n} \hat{\varphi}^\dagger(\chi_i, \xi_i)\right] |0\rangle\langle 0| \left[\prod_{i=1}^{n} \hat{\varphi}(\chi_i, \xi_i)\right]$$

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## Relational observables

P.W.-like: $\langle \hat{\Xi}_\chi \rangle_\psi \propto \langle \{\hat{\Xi}, \hat{E}_\chi\} \rangle_\psi$

- Is it a sensible definition? $\hat{E}_\chi$ is not a projector!
- ⚠ Compare with previous results when $|\psi\rangle = |\sigma\rangle$!

LM, Oriti, Wilson-Ewing (in progress).