

Emergent Cosmological Physics from Quantum Gravity

Luca Marchetti

Foundations of Observational, Classical and Semi-Classical Gravitational Physics
and The Problem of Agency and Laws of Nature
Munich, 28 March 2023

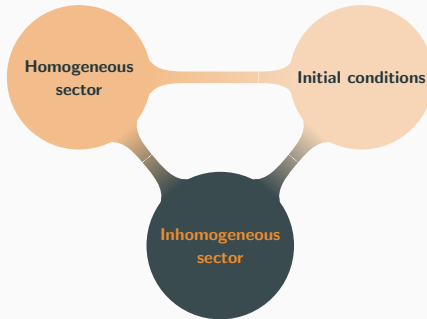
Department of Mathematics and Statistics
UNB Fredericton

Overview

- **The problem of cosmic emergence**
- **Physical localization and continuum limit in Quantum Gravity**
 - The continuum limit in Quantum Gravity
 - Relational strategy: the classical and quantum perspectives
 - Relational strategy: the quantum emergent perspective
- **Introduction to (T)GFTs**
- **Cosmology from (T)GFT**
 - (T)GFT condensates and effective relationality
 - Emergent effective Friedmann dynamics
 - Inhomogeneities, inflation, dark energy and running couplings

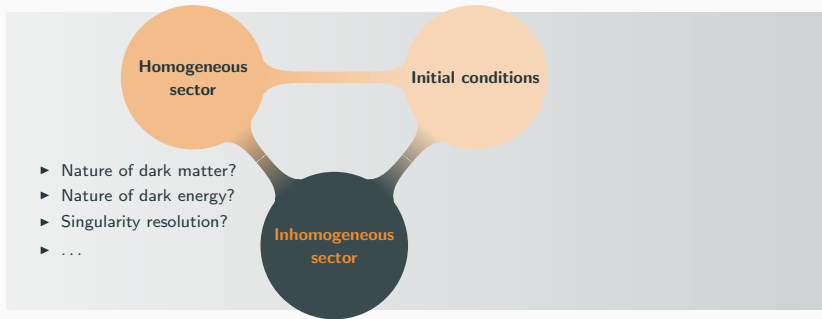
The problem of cosmic emergence

Cosmic emergence from QG: the main challenges

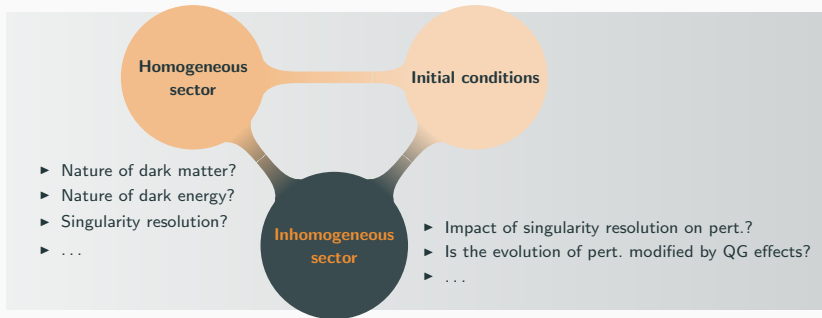


Ashtekar, Kaminski, Lewandowski 0901.0933; Agullo, Ashtekar, Nelson 1302.0254; Gielen, Oriti 1709.01095; Gerhart, Oriti, Wilson-Ewing 1805.03099; ...

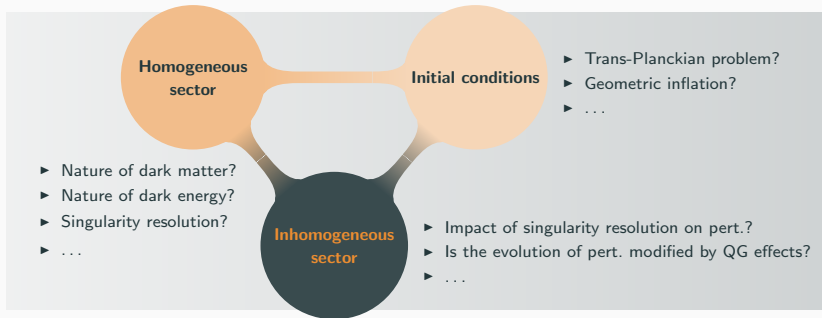
Cosmic emergence from QG: the main challenges



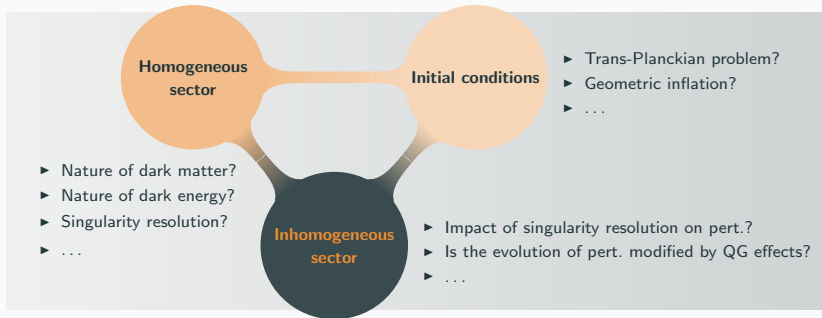
Cosmic emergence from QG: the main challenges



Cosmic emergence from QG: the main challenges

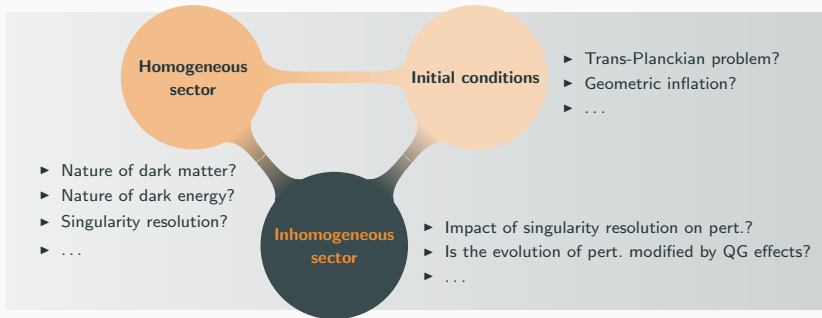


Cosmic emergence from QG: the main challenges



How to extract cosmology from quantum gravity?

Cosmic emergence from QG: the main challenges

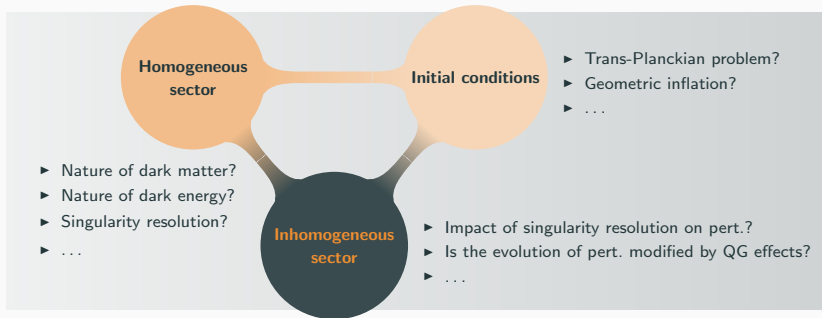


How to extract cosmology from quantum gravity?

Both technical and conceptual challenges:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?

Cosmic emergence from QG: the main challenges



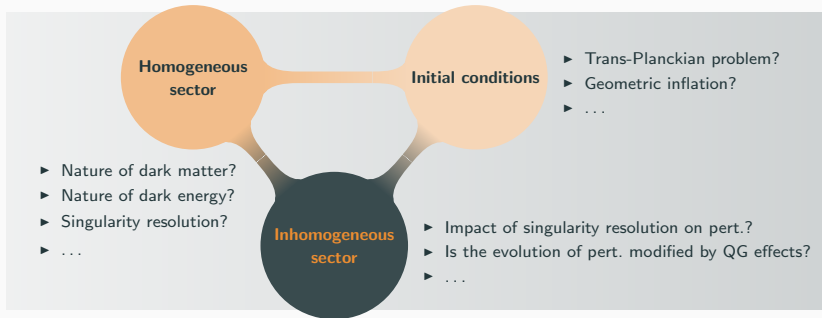
How to extract cosmology from quantum gravity?

Both technical and conceptual challenges:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?

Problem of localization

Cosmic emergence from QG: the main challenges



How to extract cosmology from quantum gravity?

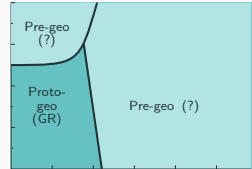
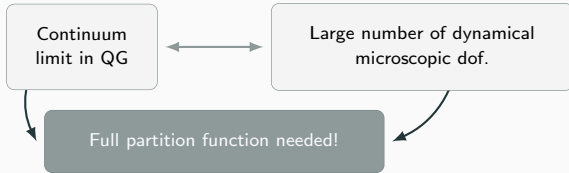
Both technical and conceptual challenges:



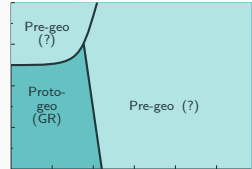
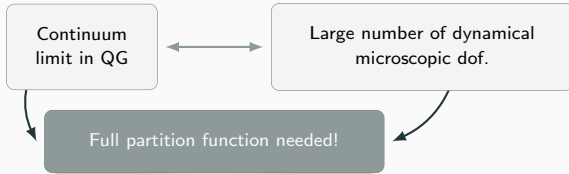
Physical localization and continuum limit in Quantum Gravity

Continuum limit in Quantum Gravity

Continuum physics and QG: the general perspective



Continuum physics and QG: the general perspective



The (F)RG perspective

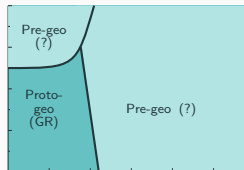
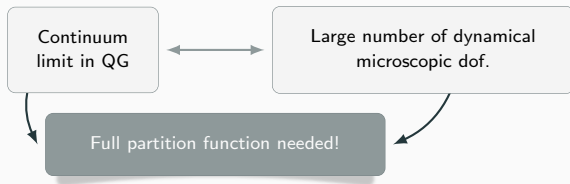
QFT on spacetime

- Energy scale defines the flow from IR and UV.

QG theory

- Only **internal** “timeless” scales available.

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

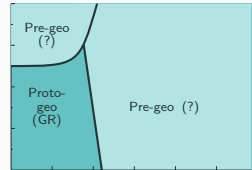
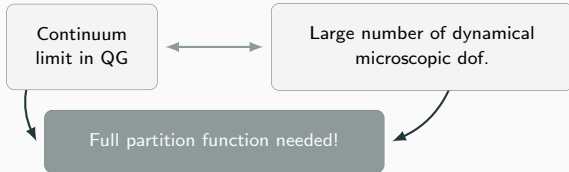
- Energy scale defines the flow from IR and UV.

QG theory

- Only **internal** “timeless” scales available.

UV and IR have different meaning in QG!

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

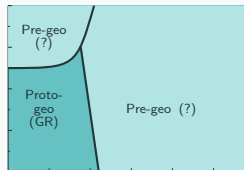
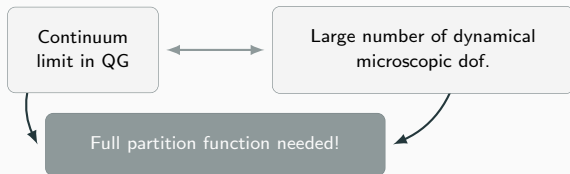
QG theory

- ▶ Only **internal** “timeless” scales available.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.
- ▶ Symmetries of QG models hard to classify.

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

QG theory

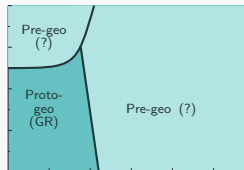
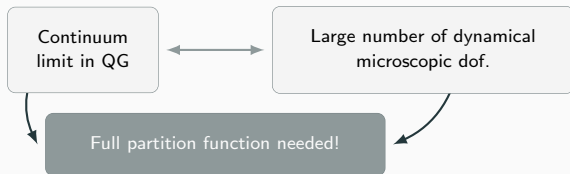
- ▶ Only **internal** "timeless" scales available.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.
- ▶ Symmetries of QG models hard to classify.

Little control over QG theory space!

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

Little control over QG theory space!

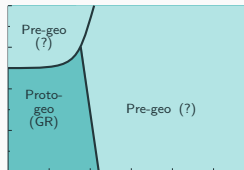
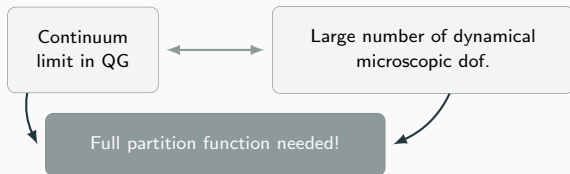
QG theory

- ▶ Only **internal** “timeless” scales available.

- ▶ Symmetries of QG models hard to classify.

Approximate methods

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

Little control over QG theory space!

QG theory

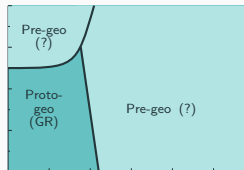
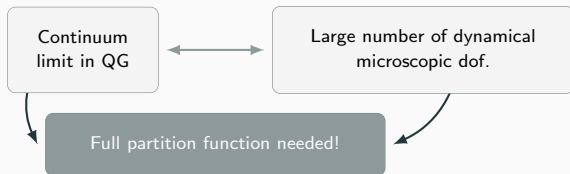
- ▶ Only **internal** “timeless” scales available.

- ▶ Symmetries of QG models hard to classify.

Approximate methods

Approximate methods

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

Little control over QG theory space!

QG theory

- ▶ Only **internal** “timeless” scales available.

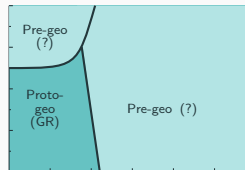
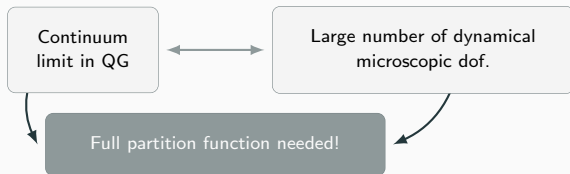
- ▶ Symmetries of QG models hard to classify.

Approximate methods

Approximate methods

Mathematical control

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

Little control over QG theory space!

QG theory

- ▶ Only **internal** “timeless” scales available.

- ▶ Symmetries of QG models hard to classify.

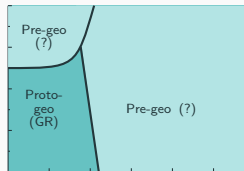
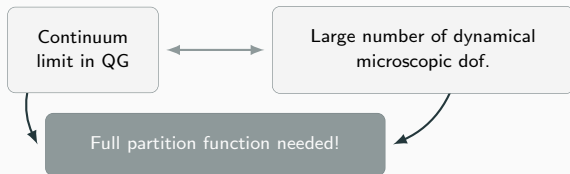
Approximate methods

Approximate methods

Mathematical control

Physical insights

Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

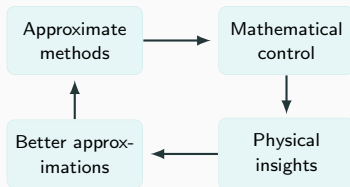
Little control over QG theory space!

QG theory

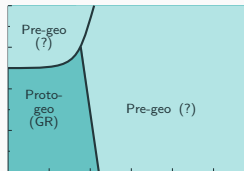
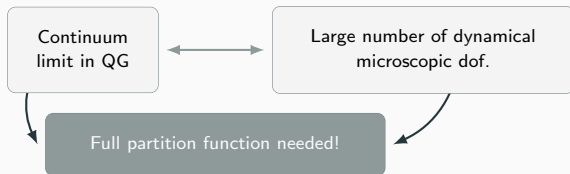
- ▶ Only **internal** “timeless” scales available.

- ▶ Symmetries of QG models hard to classify.

Approximate methods



Continuum physics and QG: the general perspective



The (F)RG perspective

QFT on spacetime

- ▶ Energy scale defines the flow from IR and UV.

UV and IR have different meaning in QG!

- ▶ Theory space constrained by symmetries.

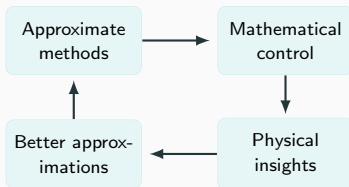
Little control over QG theory space!

QG theory

- ▶ Only **internal** “timeless” scales available.

- ▶ Symmetries of QG models hard to classify.

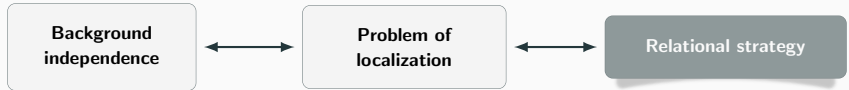
Approximate methods



Mean-field (saddle-point) approx. of Z .

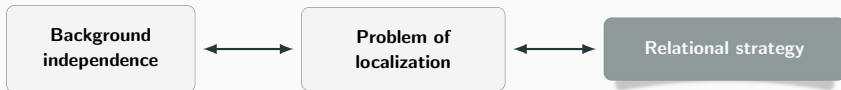
Physical localization

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Classical

Physical localization via **relational observables**:

- ▶ Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- ▶ The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- ▶ Evolution in τ is relational.
- ▶ $F_{f,T}(\tau)$ is a very complicated function.
- ▶ Applications almost only for very simple systems.

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Classical

Physical localization via **relational observables**:

- ▶ Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- ▶ The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- ▶ Evolution in τ is relational.
- ▶ $F_{f,T}(\tau)$ is a very complicated function.
- ▶ Applications almost only for very simple systems.

Quantum GR

Dirac approach: Quantize first.

- ▶ Perspective neutral.
- ▶ Poor control of the physical Hilbert space.

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Classical

Physical localization via **relational observables**:

- ▶ Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- ▶ The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- ▶ Evolution in τ is relational.
- ▶ $F_{f,T}(\tau)$ is a very complicated function.
- ▶ Applications almost only for very simple systems.

Quantum GR

Dirac approach: Quantize first.

- ▶ Perspective neutral.
- ▶ Poor control of the physical Hilbert space.

Reduced approach: Relativity first

- ▶ No quantum constraint to solve.
- ▶ Not perspective neutral. Too complicated to implement in most of the cases.

Relational strategy and emergent quantum gravity theories



A genuinely new dimension of the problem arises for **emergent** QG theories.

Relational strategy and emergent quantum gravity theories



A genuinely new dimension of the problem arises for **emergent** QG theories.

Microscopic pre-geo

- ▶ Fundamental d.o.f. are weakly related to spacetime quantities;
- ▶ The latter expected to emerge from the former when a continuum limit is taken.

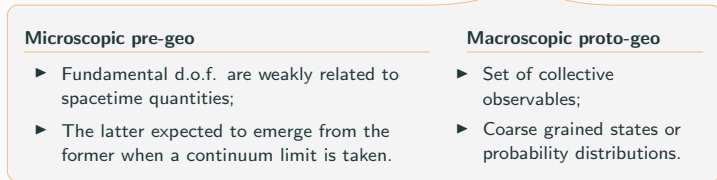
Macroscopic proto-geo

- ▶ Set of collective observables;
- ▶ Coarse grained states or probability distributions.

Relational strategy and emergent quantum gravity theories



A genuinely new dimension of the problem arises for **emergent** QG theories.

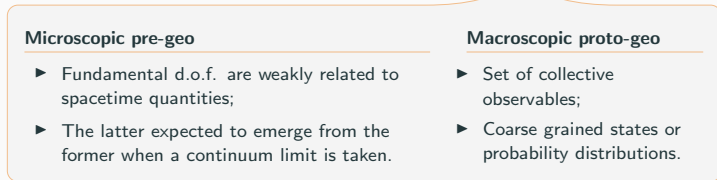


The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

Relational strategy and emergent quantum gravity theories



A genuinely new dimension of the problem arises for **emergent** QG theories.



The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

Effective approaches:

- ▶ More mathematical control and physical insights.
- ▶ Relevant for observative purposes.

Introduction to (T)GFTs

The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or $\text{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

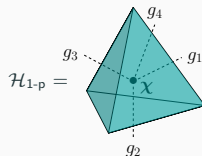
The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or $\mathrm{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:



The (T)GFT approach to quantum gravity

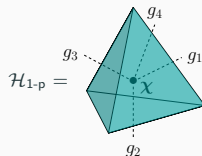
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or $\mathrm{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- Appropriate (geometricity) constraints allow the simplicial interpretation.



The (T)GFT approach to quantum gravity

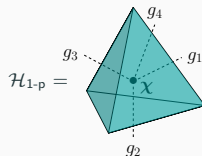
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or $\mathrm{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.
- ▶ G (Lie algebra \mathfrak{g}) variables associated to **discretized gravitational** quantities.
- ▶ $\chi^\alpha \in \mathbb{R}^{d_l} = \mathcal{M}$ associated to d_l **discretized scalar fields**.



The (T)GFT approach to quantum gravity

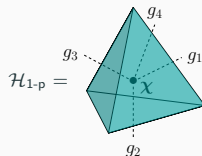
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or $\text{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.
- ▶ G (Lie algebra \mathfrak{g}) variables associated to discretized gravitational quantities.
- ▶ $\chi^\alpha \in \mathbb{R}^{d_l} = \mathcal{M}$ associated to d_l discretized scalar fields.



(T)GFT Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity (+ matter) path integral.

$$Z_{\text{GFT}} = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

The (T)GFT approach to quantum gravity

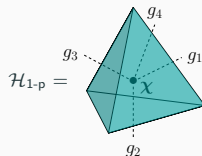
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or $\text{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.
- ▶ G (Lie algebra \mathfrak{g}) variables associated to **discretized gravitational** quantities.
- ▶ $\chi^\alpha \in \mathbb{R}^{d_l} = \mathcal{M}$ associated to d_l **discretized scalar fields**.



(T)GFT Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity (+ matter) path integral.

- ▶ **Non-local and combinatorial** interactions represent the gluing of $d - 1$ -simplices into d -simplices.
- ▶ Γ are **dual to spacetime triangulations**.

$$Z_{\text{GFT}} = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

The (T)GFT approach to quantum gravity

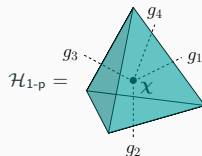
(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \times \mathcal{M} \rightarrow \mathbb{C}$

d is the dimension of the “spacetime to be” ($d = 4$);
 G local gauge group of gravity, $G = \text{SL}(2, \mathbb{C})$ or $\text{SU}(2)$; \mathcal{M} matter manifold, $\mathcal{M} = \mathbb{R}$ for scalar field.

(T)GFT Quanta

Quanta are $d - 1$ -simplices decorated with group theoretic data:

- ▶ Appropriate (geometricity) constraints allow the simplicial interpretation.
- ▶ G (Lie algebra \mathfrak{g}) variables associated to **discretized gravitational** quantities.
- ▶ $\chi^\alpha \in \mathbb{R}^{d_l} = \mathcal{M}$ associated to d_l **discretized scalar fields**.



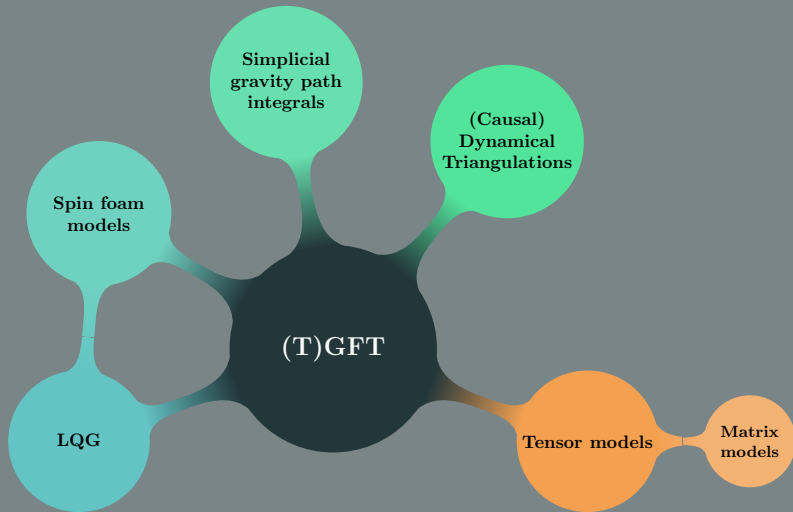
(T)GFT Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity (+ matter) path integral.

- ▶ **Non-local and combinatorial** interactions represent the gluing of $d - 1$ -simplices into d -simplices.
- ▶ Γ are **dual to spacetime triangulations**.

$$Z_{\text{GFT}} = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma}$$

GFTs are QFTs of atoms of spacetime.



Cosmology from (T)GFT

Mean-field approximation and (T)GFT condensates

(T)GFT condensates and localization

(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d \chi \int d\mathbf{g}_a \sigma(\mathbf{g}_a, \chi^\alpha) \hat{\varphi}^\dagger(\mathbf{g}_a, \chi^\alpha) \right] |0\rangle$$

(T)GFT condensates and localization

(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle$$

- ▶ Assuming $\sigma(g_a, \cdot) = \sigma(h g_a h', \cdot)$, $\mathcal{D} = \text{GL}(3)/\text{O}(3) \times \mathbb{R}^{d_I}$:
- ▶ \mathcal{D} = space of spatial geometries + matter at a point.

$\sigma(g_a, \chi^\alpha) \sim$ distribution on superspace.

(T)GFT condensates and localization

Collective states

(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\phi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle$$

- ▶ Assuming $\sigma(g_a, \cdot) = \sigma(h g_a h', \cdot)$, $\mathcal{D} = \text{GL}(3)/\text{O}(3) \times \mathbb{R}^{d_I}$:
- ▶ \mathcal{D} = space of spatial geometries + matter at a point.

$\sigma(g_a, \chi^\alpha) \sim$ distribution on superspace.

Relativity

Condensate Peaked States

- ▶ If σ is peaked on $4 \chi^\mu \simeq x^\mu$, $|\sigma\rangle_x$ encodes local info. about spatial geometry + matter at x^μ .
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- ▶ Relational localization implemented at an effective level on “hydrodynamic” (averaged) quantities.

(T)GFT condensates and localization

Collective states

(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d l \chi \int d g_a \sigma(g_a, \chi^\alpha) \hat{\varphi}^\dagger(g_a, \chi^\alpha) \right] |0\rangle$$

- ▶ Assuming $\sigma(g_a, \cdot) = \sigma(hg_a h', \cdot)$, $\mathcal{D} = \text{GL}(3)/\text{O}(3) \times \mathbb{R}^{d_I}$:
- ▶ \mathcal{D} = space of spatial geometries + matter at a point.

$\sigma(g_a, \chi^\alpha) \sim$ distribution on superspace.

Relationality

Condensate Peaked States

- ▶ If σ is peaked on $4 \chi^\mu \simeq x^\mu$, $|\sigma\rangle_x$ encodes local info. about spatial geometry + matter at x^μ .
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- ▶ Relational localization implemented at an effective level on “hydrodynamic” (averaged) quantities.

Eff. dynamics

Mean-field approximation

- ▶ QG (non-local and non-linear) counterpart of Gross-Pitaevskii eq.
- ▶ Quantum fluid description of the QG system.
- ▶ Valid only in a mesoscopic regime: large N but negligible interactions.

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_a, x^\mu)} \right\rangle_{\sigma_{x^\mu}} = 0.$$

Cosmology: macroscopic variables and effective relational dynamics

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
(\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{\Pi}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{\chi^0}} = O[\tilde{\sigma}]|_{\chi^0=\chi^0}$ **hydrodynamic**
variables: functionals of $\tilde{\sigma}$ localized at χ^0 .

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
 $(\mathcal{D} = \text{minisuperspace} + \text{clock})$

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\bar{g}_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{H}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$ **hydrodynamic**
variables: functionals of $\tilde{\sigma}$ localized at x^0 .

Relationality

► Averaged evolution wrt x^0 is physical:

$$\text{Intensive} \longleftarrow \langle \hat{X} \rangle_{\sigma_{x^0}} \equiv \langle \hat{X} \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{H}^0 \rangle_{\sigma_{x^0}}$.

Macroscopic cosmological variables and effective relationality

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0
 (\mathcal{D} = minisuperspace + clock)

Collective Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 dg_a$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

$$\hat{H}^0 = -i(\hat{\varphi}^\dagger, \partial_0 \hat{\varphi})$$

► Observables \leftrightarrow collective operators on Fock space.

► $\langle \hat{O} \rangle_{\sigma_{x^0}} = O[\tilde{\sigma}]|_{\chi^0=x^0}$ **hydrodynamic**
variables: functionals of $\tilde{\sigma}$ localized at x^0 .

Relationality

► Averaged evolution wrt x^0 is physical:

$$\langle \hat{X} \rangle_{\sigma_{x^0}} \equiv \langle \hat{X} \rangle_{\sigma_{x^0}} / \langle \hat{N} \rangle_{\sigma_{x^0}} \simeq x^0$$

► Emergent effective relational description:

- Small clock quantum fluctuations.
- Effective Hamiltonian $H_{\sigma_{x^0}} \simeq \langle \hat{H}^0 \rangle_{\sigma_{x^0}}$.

Wavefunction
 $\xrightarrow{\text{isotropy}}$

$$\langle \hat{V} \rangle_{\sigma_x^0} = \sum_v v_v |\tilde{\sigma}_v|^2(x^0)$$

$$\langle \hat{N} \rangle_{\sigma_x^0} = \sum_v |\tilde{\sigma}_v|^2(x^0)$$

Emergent cosmological physics

Mean-field approximation

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel.
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0\tilde{\sigma}_v' - E_v^2\tilde{\sigma} = 0.$$

Effective relational volume dynamics

Mean-field approximation

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel.
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0\tilde{\sigma}_v' - E_v^2\tilde{\sigma} = 0.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\oint_v V_v \rho_v \text{sgn}(\rho_v') \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3\oint_v V_v \rho_v^2}\right)^2 \quad \frac{V''}{V} \simeq \frac{2\oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Effective relational volume dynamics

Mean-field approximation

- Mesoscopic regime: large N but negligible interactions.
- Hydrodynamic approx. of kinetic kernel.
- Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0\tilde{\sigma}_v' - E_v^2\tilde{\sigma} = 0.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\oint_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3\oint_v V_v \rho_v^2}\right)^2 \quad \frac{V''}{V} \simeq \frac{2\oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Classical limit (large ρ_v s, late times)

- If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- Quantum fluctuations on clock and geometric variables are under control.

Effective relational volume dynamics

Mean-field approximation

- Mesoscopic regime: large N but negligible interactions.
- Hydrodynamic approx. of kinetic kernel.
- Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0\tilde{\sigma}_v' - E_v^2\tilde{\sigma} = 0.$$

Effective relational Friedmann dynamics

$$\left(\frac{V'}{3V}\right)^2 \simeq \left(\frac{2\oint_v V_v \rho_v \text{sgn}(\rho_v') \sqrt{\mathcal{E}_v - Q_v^2/\rho_v^2 + \mu_v^2 \rho_v^2}}{3\oint_v V_v \rho_v^2}\right)^2 \quad \frac{V''}{V} \simeq \frac{2\oint_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\oint_v V_v \rho_v^2}$$

Classical limit (large ρ_v s, late times)

- If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- Quantum fluctuations on clock and geometric variables are under control.

Bounce

- A non-zero volume bounce happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

Geometric acceleration from interactions

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.
- ⚠ Insights on renormalization?
- ⚠ Connection with asymptotic safety?

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.
- ⚠ Insights on renormalization?
- ⚠ Connection with asymptotic safety?

Effective dynamics of cosmological scalar perturbations

- ▶ Small inhomogeneities at the level of σ .
- ✓ Only late times super-horizon matching with GR.
- ✓ Near-bounce super-horizon behavior different even from modified gravity.

Physics of (T)GFT cosmology

Including interactions

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.
- ⚠ Insights on renormalization?
- ⚠ Connection with asymptotic safety?

Inhomogeneities

Effective dynamics of cosmological scalar perturbations

- ▶ Small inhomogeneities at the level of σ .
- ✓ Only late times super-horizon matching with GR.
- ✓ Near-bounce super-horizon behavior different even from modified gravity.

Why the mismatch?

Physics of (T)GFT cosmology

Including interactions

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.
- ⚠ Insights on renormalization?
- ⚠ Connection with asymptotic safety?

Inhomogeneities

Effective dynamics of cosmological scalar perturbations

- ▶ Small inhomogeneities at the level of σ .
- ✓ Only late times super-horizon matching with GR.
- ✓ Near-bounce super-horizon behavior different even from modified gravity.
- ⚠ Full relational frame requires quanta with different causal properties.
- ⚠ Including quantum correlations seems to substantially help the matching.

Why the mismatch?

Physics of (T)GFT cosmology

Including interactions

Geometric acceleration from interactions

Early times: geometric inflation

- ✓ Geometric inflation from QG interactions.
- ⚠ For some models bottom-up natural and slow-roll.
- ⚠ Comparison with observations?

Late times: phantom dark energy

- ✓ Phantom dark energy generated by QG effects (no field theoretic issue).
- ⚠ Comparison with observations?

Including more realistic matter: running couplings

- ✓ Matching with GR requires the macroscopic constants (including G) to run with time.
- ⚠ Insights on renormalization?
- ⚠ Connection with asymptotic safety?

Inhomogeneities

Effective dynamics of cosmological scalar perturbations

- ▶ Small inhomogeneities at the level of σ .
- ✓ Only late times super-horizon matching with GR.
- ✓ Near-bounce super-horizon behavior different even from modified gravity.
- ⚠ Full relational frame requires quanta with different causal properties.
- ⚠ Including quantum correlations seems to substantially help the matching.

Why the mismatch?

Geometry from quantum correlations!

microscopic QG model

- ▶ What are the fundamental QG degrees of freedom?

microscopic QG model

- ▶ (T)GFT: simplices decorated with discretized fields.

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain?

Mean-field approx. (condensate states).



- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain?

Mean-field approx. (condensate states).



effective description

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

How to make it relational? Use physically localized states (effective only).

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

How to make it relational? Use physically localized states (effective only).

macroscopic relational physics

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

How to make it relational? Use physically localized states (effective only).

macroscopic relational physics

Homogeneous, free

- Late times: FRLW flat classical dynamics.
- Early times: averaged quantum bounce.

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

How to make it relational? Use physically localized states (effective only).

macroscopic relational physics

Homogeneous, free

- Late times: FRLW flat classical dynamics.
- Early times: averaged quantum bounce.

Homogeneous, interacting

- Late times: emergence of phantom dark energy.
- Early times: possible geometric slow-roll inflation.

- What are the fundamental QG degrees of freedom?

microscopic QG model

- (T)GFT: simplices decorated with discretized fields.

How to coarse grain? Mean-field approx. (condensate states).

- What are the macroscopic variables?

effective description

- Superspace σ -averages of collective observables.

How to make it relational? Use physically localized states (effective only).

macroscopic relational physics

Homogeneous, free

- Late times: FRLW flat classical dynamics.
- Early times: averaged quantum bounce.

Homogeneous, interacting

- Late times: emergence of phantom dark energy.
- Early times: possible geometric slow-roll inflation.

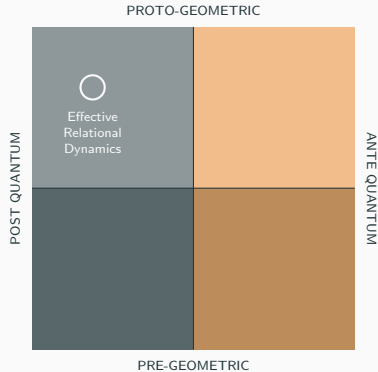
(Scalar) Inhomogeneities, free

- Late times: indications for GR matching at all scales.
- Early times: deviations from classical and modified gravity.

Backup



Emergent effective relational dynamics

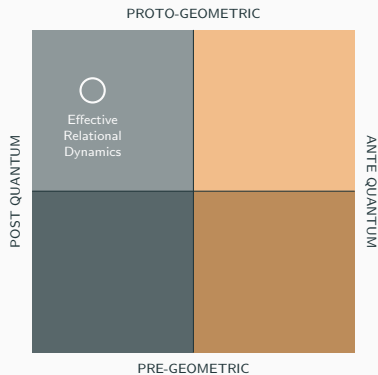


Basic principles

Emergence Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

Effectiveness Rel. evolution intended to hold on average. Internal clock not too quantum.

Emergent effective relational dynamics



Basic principles

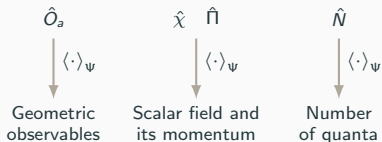
Emergence Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

Effectiveness Rel. evolution intended to hold on average. Internal clock not too quantum.

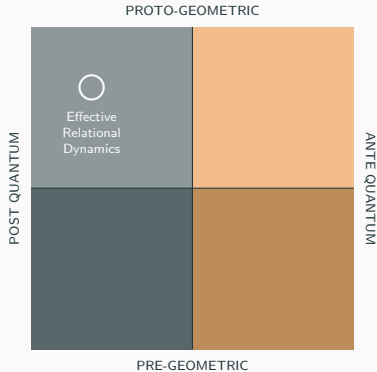
Concrete example: scalar field clock

Emergence

- Identify a class of states $|\Psi\rangle$ which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- Identify a set of collective observables:



Emergent effective relational dynamics



Basic principles

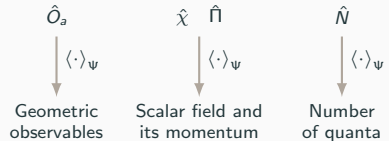
Emergence Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

Effectiveness Rel. evolution intended to hold on average. Internal clock not too quantum.

Concrete example: scalar field clock

Emergence

- Identify a class of states $|\Psi\rangle$ which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- Identify a set of collective observables:



Effectiveness

- It exists a “Hamiltonian” \hat{H} such that

$$i \frac{d}{d \langle \hat{\chi} \rangle_\Psi} \langle \hat{O}_a \rangle_\Psi = \langle [\hat{H}, \hat{O}_a] \rangle_\Psi,$$

and whose moments coincide with those of $\hat{\Pi}$.

- Relative variance of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$ and have the characteristic $\langle \hat{N} \rangle_\Psi^{-1}$ behavior:

$$\sigma_\chi^2 \ll 1, \quad \sigma_\chi^2 \sim \langle \hat{N} \rangle_\Psi^{-1}.$$

The (T)GFT approach to QG



GFTs are QFTs of atoms of spacetime.

- ▶ Take seriously the idea of a microscopic structure of spacetime.
- ▶ Related to canonical and discrete path-integral approaches to QG.
- ▶ Physical insights from canonical approaches combined with powerful field theoretic methods!

The (T)GFT approach to QG

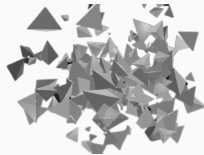


GFTs are QFTs of atoms of spacetime.

- ▶ Take seriously the idea of a microscopic structure of spacetime.
- ▶ Related to canonical and discrete path-integral approaches to QG.
- ▶ Physical insights from canonical approaches combined with powerful field theoretic methods!

Group Field Theory Quanta

- ▶ GFT quanta are atoms of quantum spacetime, i.e. $d - 1$ -dimensional simplices.
- ▶ Data associated to a single quantum are geometric data of a $d - 1$ -simplex.



The (T)GFT approach to QG

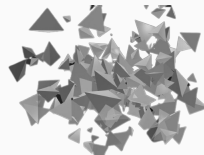


GFTs are QFTs of atoms of spacetime.

- ▶ Take seriously the idea of a microscopic structure of spacetime.
- ▶ Related to canonical and discrete path-integral approaches to QG.
- ▶ Physical insights from canonical approaches combined with powerful field theoretic methods!

Group Field Theory Quanta

- ▶ GFT quanta are atoms of quantum spacetime, i.e. $d - 1$ -dimensional simplices.
- ▶ Data associated to a single quantum are geometric data of a $d - 1$ -simplex.



Group Field Theory Processes

- ▶ GFT Feynman diagrams (QG processes) are associated to d -dimensional triangulated manifolds.
- ▶ Data associated to QG processes are geometric data of d -dimensional triangulated manifolds.

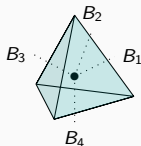


Oriti 0912.2441; Oriti 1110.5606; Oriti 1408.7112; Krajewski 1210.6257; Oriti 1807.04875; Gielen, Sindoni 1602.08104; ...

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.

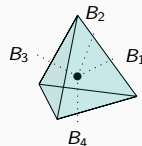


Group Field Theory Quanta

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

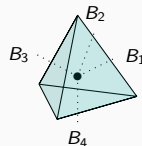
- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Group Field Theory Quanta

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Lie algebra rep.

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{spin}(4))$$

Group Field Theory Quanta

Atoms of spacetime

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Reps.

Lie algebra rep.
 $\mathcal{H}_{\Delta_a} = L^2(\mathfrak{spin}(4))$

Non-comm.
FT

Lie group rep.
 $\mathcal{H}_{\Delta_a} = L^2(\text{Spin}(4))$

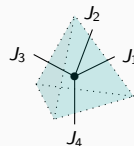
Group Field Theory Quanta

Atoms of spacetime

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Reps.

$$\begin{array}{ccccc}
 \text{Lie algebra rep.} & \xleftrightarrow[\text{FT}]{\text{Non-comm.}} & \text{Lie group rep.} & \xleftrightarrow[\text{Theorem}]{\text{Peter-Weyl}} & \text{Spin rep.} \\
 \mathcal{H}_{\Delta_a} = L^2(\mathfrak{spin}(4)) & & \mathcal{H}_{\Delta_a} = L^2(\text{Spin}(4)) & & \mathcal{H}_{\Delta_a} = \bigoplus_{J_a} \mathcal{H}_{J_a}
 \end{array}$$

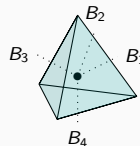
Group Field Theory Quanta

Atoms of spacetime

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Reps.

Lie algebra rep.

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{spin}(4))$$

Non-comm.

FT

Lie group rep.

$$\mathcal{H}_{\Delta_a} = L^2(\text{Spin}(4))$$

Peter-Weyl

Theorem

Spin rep.

$$\mathcal{H}_{\Delta_a} = \bigoplus_{J_a} \mathcal{H}_{J_a}$$

Gravity

Discretized gravity

- Discretized Palatini gravity can be written as constrained BF theory.
- $B \sim e \wedge e$ and $g \sim \mathcal{P} \exp \omega$.

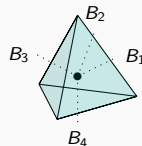
Group Field Theory Quanta

Atoms of spacetime

Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- **Closure:** $\sum_a B_a = 0$ (faces of the tetrahedron close).
- **Simplicity:** $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\text{Spin}(4)) \sim \text{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{\text{tetra}} \subset \bigotimes_{a=1}^4 \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

Reps.

Lie algebra rep.

$$\mathcal{H}_{\Delta_a} = L^2(\mathfrak{spin}(4))$$

Non-comm.

FT

Lie group rep.

$$\mathcal{H}_{\Delta_a} = L^2(\text{Spin}(4))$$

Peter-Weyl

Theorem

Spin rep.

$$\mathcal{H}_{\Delta_a} = \bigoplus_{J_a} \mathcal{H}_{J_a}$$

Gravity

Discretized gravity

- Discretized Palatini gravity can be written as constrained BF theory.
- $B \sim e \wedge e$ and $g \sim \mathcal{P} \exp \omega$.

Loop Quantum Gravity

- Fix the normal and reduce to $\text{SU}(2)$.

$$\mathcal{H}_{\text{tetra}} = \text{open spin-network space} = \bigoplus_{\vec{J}} \left[\bigotimes_{a=1}^4 \mathcal{H}_{J_a} \otimes \mathcal{I}^{\vec{J}} \right]$$

A many-body theory for spacetime atoms

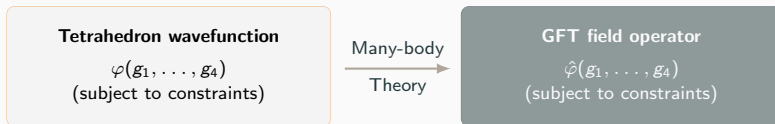
Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

Oriti 1310.7786; Gielen, Oriti 1407.8167; Gielen, Sindoni 1602.08104; Oriti, Sindoni, Wilson-Ewing 1602.05881; ...

A many-body theory for spacetime atoms



Oriti 1310.7786; Gielen, Oriti 1407.8167; Gielen, Sindoni 1602.08104; Oriti, Sindoni, Wilson-Ewing 1602.05881; ...

A many-body theory for spacetime atoms

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4) \\ \text{(subject to constraints)}$$

Many-body
Theory \longrightarrow

GFT field operator

$$\hat{\varphi}(g_1, \dots, g_4) \\ \text{(subject to constraints)}$$

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic quantum states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.

GFT Fock space

A many-body theory for spacetime atoms

Tetrahedron wavefunction

$$\varphi(g_1, \dots, g_4)$$

(subject to constraints)

Many-body
Theory \longrightarrow

GFT field operator

$$\hat{\varphi}(g_1, \dots, g_4)$$

(subject to constraints)

$$\mathcal{F}_{\text{GFT}} = \bigoplus_{V=0}^{\infty} \text{sym} \left[\mathcal{H}_{\text{tetra}}^{(1)} \otimes \mathcal{H}_{\text{tetra}}^{(2)} \otimes \dots \mathcal{H}_{\text{tetra}}^{(V)} \right]$$

- ▶ \mathcal{F}_{GFT} generated by action of $\hat{\varphi}^\dagger(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^\dagger(g'_a)] = \mathbb{I}_G(g_a, g'_a)$.
- ▶ $\mathcal{H}_\Gamma \subset \mathcal{F}_{\text{GFT}}$, \mathcal{H}_Γ space of states associated to connected simplicial complexes Γ .
- ▶ Generic quantum states **do not** correspond to connected simplicial lattices nor classical simplicial geometries.

Volume operator $V = \int dg_a^{(1)} dg_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^\dagger(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{J_a} V_{J_a} \hat{\varphi}_{J_a}^\dagger \hat{\varphi}_{J_a}$.

- ▶ Generic second quantization prescription to build a $m + n$ -body operator: sandwich matrix elements between spin-network states between m powers of $\hat{\varphi}^\dagger$ and n powers of $\hat{\varphi}$.

GFT Fock space

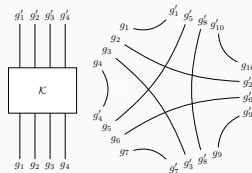
Operators

Group Field Theory dynamics

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\gamma}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



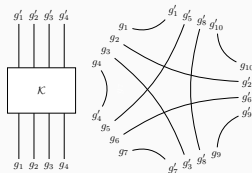
Group Field Theory dynamics

GFT action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\gamma}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} .$$

- Diagrams Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.

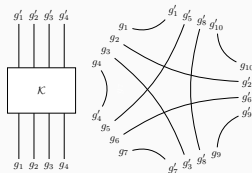
Group Field Theory dynamics

GFT action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\gamma}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{spinfoam model} .$$

- Diagrams Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V} can be chosen to match the desired simplicial gravity path-integral.

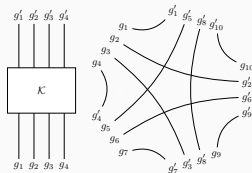
Group Field Theory dynamics

GFT action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\gamma}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{spinfoam model} .$$

- Diagrams Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V} can be chosen to match the desired simplicial gravity path-integral.

Example

Boulatov model: $g_a \in \text{SU}(2)$, $a = 1, 2, 3$, $\mathcal{K} = \delta(g_a, g_b)$, $\gamma = \triangle$.

3d gravity
on lattice
dual to Γ

$$\int \prod_{\ell} dg_{\ell} \prod_f \delta\left(\prod_{\ell \in \partial f} g_{\ell}\right) = A_{\Gamma}$$

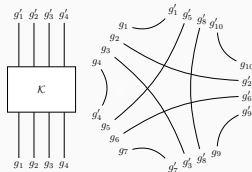
Group Field Theory dynamics

GFT action

$$S[\varphi, \bar{\varphi}] = \int dg_a \bar{\varphi}(g_a) \mathcal{K}[\varphi](g_a) + \sum_{\gamma} \frac{\lambda_{\gamma}}{n_{\gamma}} \text{Tr}_{\gamma}[\varphi] + \text{c.c.} .$$

- Interaction terms are **combinatorially non-local**.
- Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ :

$$\text{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} dg_a \prod_{(a,i;b,j)} \mathcal{V}(g_a^{(i)}, g_b^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_a^{(i)}) .$$



Partition function

$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\}) A_{\Gamma} = \text{spinfoam model} .$$

- Diagrams Γ = stranded diagrams dual to d -dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.
- \mathcal{K} and \mathcal{V} can be chosen to match the desired simplicial gravity path-integral.

Example

Boulatov model: $g_a \in \text{SU}(2)$, $a = 1, 2, 3$, $\mathcal{K} = \delta(g_a, g_b)$, $\gamma = \triangle$.

3d gravity
on lattice
dual to Γ

$$\int \prod_{\ell} dg_{\ell} \prod_f \delta\left(\prod_{\ell \in \partial f} g_{\ell}\right) = A_{\Gamma} = \sum_{j_e} \prod_e d_e \prod_{\tau} \left\{ \begin{matrix} j_1^{\tau} & j_2^{\tau} & j_3^{\tau} \\ j_4^{\tau} & j_5^{\tau} & j_6^{\tau} \end{matrix} \right\}$$

Spinfoam ampli-
tude on lattice
dual to Γ

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int dh_a \int d\chi \mathcal{K}(g_a, h_a, (x^0 - \chi)^2) \sigma_{x^0}(h_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^0)} \Big|_{\varphi=\sigma_{x^0}} = 0.$$

- **Non-localities** present in geometric (g_a) and pre-matter (χ) variables.
- **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_a, x^0)} \right\rangle_{\sigma, x^0} = \int dh_a \int d\chi \mathcal{K}(g_a, h_a, (x^0 - \chi)^2) \sigma_{x^0}(h_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^0)} \Big|_{\varphi = \sigma_{x^0}} = 0.$$

- **Non-localities** present in geometric (g_a) and pre-matter (χ) variables.
- **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_a, x^0)} \right\rangle_{\sigma, x^0} = \int dh_a \int d\chi \mathcal{K}(g_a, h_a, (x^0 - \chi)^2) \sigma_{x^0}(h_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^0)} \Big|_{\varphi = \sigma_{x^0}} = 0.$$

- ▶ **Non-localities** present in geometric (g_a) and pre-matter (χ) variables.
- ▶ **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

- ▶ Only statistical interpretation of σ , as a distribution producing observable averages.

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (x^0 - \chi)^2) \sigma_{x^0}(\mathbf{h}_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, x^0)} \Big|_{\varphi=\sigma_{x^0}} = 0.$$

- **Non-localities** present in geometric (\mathbf{g}_a) and pre-matter (χ) variables.
- **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

- Only statistical interpretation of σ , as a distribution producing observable averages.

Working approximations

- Mesoscopic regime: large N but negligible interactions.
- Linearized dynamics.

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (x^0 - \chi)^2) \sigma_{x^0}(\mathbf{h}_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, x^0)} \Big|_{\varphi=\sigma_{x^0}} = 0.$$

- ▶ **Non-localities** present in geometric (\mathbf{g}_a) and pre-matter (χ) variables.
- ▶ **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

- ▶ Only statistical interpretation of σ , as a distribution producing observable averages.

Working approximations

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Linearized dynamics.
- ▶ Hydrodynamic truncation of kinetic kernel due to peaking properties.
- ▶ Differential equation in x^0 .

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int dh_a \int d\chi \mathcal{K}(g_a, h_a, (x^0 - \chi)^2) \sigma_{x^0}(h_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^0)} \Big|_{\varphi=\sigma_{x^0}} = 0.$$

- ▶ **Non-localities** present in geometric (g_a) and pre-matter (χ) variables.
- ▶ **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

- ▶ Only statistical interpretation of σ , as a distribution producing observable averages.

Working approximations

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic truncation of kinetic kernel due to peaking properties.
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables, with
 - $v = j \in \mathbb{N}/2$ for SU(2) (EPRL-like);
 - $v = \rho \in \mathbb{R}$ for SL(2, \mathbb{C}) (extended BC).
- ▶ Linearized dynamics.
- ▶ Differential equation in x^0 .
- ▶ Localization wrt. v (in EPRL and extended BC models).

Mean-field approximation and QG hydrodynamics

Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int d\mathbf{h}_a \int d\chi \mathcal{K}(\mathbf{g}_a, \mathbf{h}_a, (x^0 - \chi)^2) \sigma_{x^0}(\mathbf{h}_a, \chi) + \lambda \left. \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(\mathbf{g}_a, x^0)} \right|_{\varphi=\sigma_{x^0}} = 0.$$

- ▶ **Non-localities** present in geometric (\mathbf{g}_a) and pre-matter (χ) variables.
- ▶ **Non-linearities** prevent any quantum-mechanical interpretation for σ (no superposition principle).

$\sigma \neq$ Wavefunction of the Universe (though they share the same domain)

- ▶ Only statistical interpretation of σ , as a distribution producing observable averages.

Working approximations

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic truncation of kinetic kernel due to peaking properties.
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables, with
 - $v = j \in \mathbb{N}/2$ for SU(2) (EPRL-like);
 - $v = \rho \in \mathbb{R}$ for SL(2, \mathbb{C}) (extended BC).
- ▶ Linearized dynamics.
- ▶ Differential equation in x^0 .
- ▶ Localization wrt. v (in EPRL and extended BC models).

$$\tilde{\sigma}_v'' - 2i\tilde{\pi}_0 \tilde{\sigma}_v' - E_v^2 \tilde{\sigma} = 0$$

(T)GFT and matter: scalar fields

(Tensorial) Group Field Theories:

theories of a field $\varphi : G^d \times \mathbb{R}^d \rightarrow \mathbb{C}$
defined on the product of G^d and \mathbb{R}^d .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in many applications, $G = \text{SU}(2)$.

Kinematics

Boundary states are $d - 1$ -simplices decorated with quantum geometric and scalar data:

(T)GFT and matter: scalar fields

(Tensorial) Group Field Theories:

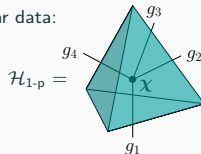
theories of a field $\varphi : G^d \times \mathbb{R}^d \rightarrow \mathbb{C}$
defined on the product of G^d and \mathbb{R}^d .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in many applications, $G = \text{SU}(2)$.

Kinematics

Boundary states are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- **Geometricity constraints** imposed analogously as before.
- Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



(T)GFT and matter: scalar fields

(Tensorial) Group Field Theories:

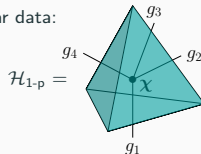
theories of a field $\varphi : G^d \times \mathbb{R}^d \rightarrow \mathbb{C}$ defined on the product of G^d and \mathbb{R}^d .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in many applications, $G = \text{SU}(2)$.

Kinematics

Boundary states are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- **Geometricity constraints** imposed analogously as before.
- Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^d$.



Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- Geometric data enter the action in a **non-local and combinatorial** fashion.
- Scalar field data are **local** in interactions.
- For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_a, g_b; \chi, \chi') = \mathcal{K}(g_a, g_b; |\chi - \chi'|^2)$$

$$\mathcal{V}(g_a^{(1)}, \dots, g_a^{(5)}, \chi) = \mathcal{V}(g_a^{(1)}, \dots, g_a^{(5)})$$