

(T)GFT and Emergent Cosmology

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Department of Mathematics and Statistics UNB Fredericton

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The (T)GFT approach to QG



GFTs are QFTs of atoms of spacetime.

- Take seriously the idea of a microscopic structure of spacetime.
- ▶ Related to canonical and discrete path-integral approaches to QG.
- Physical insights from canonical approaches combined with powerful field theoretic methods!

Oriti 0912.2441; Oriti 1110.5606; Oriti 1408.7112; Krajewski 1210.6257; Oriti 1807.04875; Gielen, Sindoni 1602.08104; ...

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Group Field Theory Quanta

- ▶ GFT quanta are atoms of quantum spacetime, i.e. d − 1-dimensional simplices.
- Data associated to a single quantum are geometric data of a d - 1-simplex.



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Group Field Theory Processes

- GFT Feynman diagrams (QG processes) are associated to d-dimensional triangulated manifolds.
- Data associated to QG processes are geometric data of d-dimensional triangulated manifolds.









Classical tetrahedron

A Euclidean tetrahedron is described by 4 bivectors $B_a \in \wedge^2 R^4$, with

- Closure: $\sum_{a} B_{a} = 0$ (faces of the tetrahedron close).
- Simplicity: $X \cdot \star B_a = 0$, i.e. B_a is simple: $(B \sim e \wedge e')$.



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B_3 B_1 B_4

Quantum tetrahedron

- Using $\wedge^2 \mathbb{R}^4 \simeq \mathfrak{spin}(4)$ the face phase space is $\mathcal{T}^*(\mathrm{Spin}(4)) \sim \mathrm{Spin}(4) \times \mathfrak{spin}(4)$.
- $\mathcal{T}^*(\text{Spin}(4))$ has a natural Poisson structure which can be canonically quantized.
- $\mathcal{H}_{tetra} \subset \otimes_{a=1}^{4} \mathcal{H}_{\Delta_a}$ (subset defined by the imposition of constraints).

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Discretized gravity

- Discretized Palatini gravity can be written as constrained BF theory.
 - $\blacktriangleright \quad B \sim e \wedge e \text{ and } g \sim \mathcal{P} \exp \omega.$

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(T)GFT and Emergent Cosmology

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Loop Quantum Gravity

► Fix the normal and reduce to SU(2).

$$\mathcal{H}_{\mathsf{tetra}} = \mathsf{open} \; \mathsf{spin-network} \; \mathsf{space} = igoplus_{ec{j}} \left[igotimes_{a=1}^4 \mathcal{H}_{j_a} \otimes \mathcal{I}^{ec{j}}
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Gravity

Tetrahedron wavefunction

 $\varphi(g_1, \ldots, g_4)$ (subject to constraints)

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$$\mathcal{F}_{\mathsf{GFT}} = \bigoplus_{V=0}^{\infty} \operatorname{sym} \left[\mathcal{H}_{\mathsf{tetra}}^{(1)} \otimes \mathcal{H}_{\mathsf{tetra}}^{(2)} \otimes \ldots \mathcal{H}_{\mathsf{tetra}}^{(V)} \right]$$

- ▶ $\mathcal{F}_{\mathsf{GFT}}$ generated by action of $\hat{\varphi}^{\dagger}(g_a)$ on $|0\rangle$, with $[\hat{\varphi}(g_a), \hat{\varphi}^{\dagger}(g_a')] = \mathbb{I}_{\mathcal{G}}(g_a, g_a')$.
- $\blacktriangleright \ \mathcal{H}_{\Gamma} \subset \mathcal{F}_{\mathsf{GFT}}, \ \mathcal{H}_{\Gamma} \ \text{space of states associated to connected simplicial complexes } \Gamma.$
- Generic quantum states do not correspond to connected simplicial lattices nor classical simplicial geometries.

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Volume operator
$$V = \int \mathrm{d}g_a^{(1)} \, \mathrm{d}g_a^{(2)} V(g_a^{(1)}, g_a^{(2)}) \hat{\varphi}^{\dagger}(g_a^{(1)}) \hat{\varphi}(g_a^{(2)}) = \sum_{J_a} V_{J_a} \hat{\varphi}^{\dagger}_{J_a} \hat{\varphi}_{J_a} \hat{\varphi}_{J_a} \hat{\varphi}_{J_a}.$$

Generic second quantization prescription to build a *m* + *n*-body operator: sandwich matrix elements between spin-network states between *m* powers of *φ*[↑] and *n* powers of *φ*^ˆ.

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Operators

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Interaction terms are combinatorially non-local.

Field arguments convoluted pairwise following the combinatorial pattern dictated by the graph γ:

 $\mathcal{S}[arphi,ar{arphi}] = \int \mathrm{d} g_{\mathfrak{s}} ar{arphi}(g_{\mathfrak{s}}) \mathcal{K}[arphi](g_{\mathfrak{s}}) + \sum_{\gamma} rac{\lambda_{\gamma}}{n_{\gamma}} \operatorname{Tr}_{\gamma}[arphi] + ext{c.c.} \,.$

$$\mathsf{Tr}_{\gamma}[\varphi] = \int \prod_{i=1}^{n_{\gamma}} \mathrm{d}g_{a} \prod_{(a,i;b,j)} \mathcal{V}(g_{a}^{(i)}, g_{b}^{(j)}) \prod_{i=1}^{n_{\gamma}} \varphi(g_{a}^{(i)})$$



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$$Z[\varphi, \bar{\varphi}] = \sum_{\Gamma} w_{\Gamma}(\{\lambda_{\gamma}\})A_{\Gamma}.$$

- · Diagrams Γ = stranded diagrams dual to *d*-dimensional cellular complexes of arbitrary topology.
- Amplitudes A_{Γ} = sums over group theoretic data associated to the cellular complex.

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Partition function

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Boulatov model:
$$g_a \in SU(2)$$
, $a = 1, 2, 3$, $\mathcal{K} = \delta(g_a, g_b)$, $\gamma = \underline{/ \cdot \cdot}$.

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Partition function

Example

(T)GFT and Emergent Cosmology

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(T)GFT and Emergent Cosmology

(Tensorial) Group Field Theories: theories of a field $\varphi : G^d \to \mathbb{C}$ defined on *d* copies of a group manifold *G*. $\begin{array}{l} d \mbox{ is the dimension of the "spacetime to be" } (d=4) \\ \mbox{ and } G \mbox{ is the local gauge group of gravity,} \\ G={\rm SL}(2,\mathbb{C}) \mbox{ or, in many applications, } G={\rm SU}(2). \end{array}$

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Kinematics

Boundary states are d - 1-simplices decorated with quantum geometric and scalar data:

- Geometricity constraints imposed analogously as before.
- Scalar field discretized on each *d*-simplex: each *d* − 1-simplex composing it carries values *x* ∈ ℝ^d.



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(Tensorial) Group Field Theories: theories of a field $\varphi : G^d \times \mathbb{R}^{d_1} \to \mathbb{C}$ defined on the product of G^d and \mathbb{R}^{d_1} . d is the dimension of the "spacetime to be" (d = 4)and G is the local gauge group of gravity, $G = SL(2, \mathbb{C})$ or, in many applications, G = SU(2).

Kinematics

Boundary states are d - 1-simplices decorated with quantum geometric and scalar data:

- Geometricity constraints imposed analogously as before.
- Scalar field discretized on each *d*-simplex: each *d* − 1-simplex composing it carries values *x* ∈ ℝ^d.

Dynamics

 S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- Geometric data enter the action in a non-local and combinatorial fashion.
- Scalar field data are local in interactions.
- ▶ For minimally coupled, free, massless scalars:





Intermezzo: the QG perspective on Cosmology

The QG perspective on Cosmology



Ashtekar, Kaminski, Lewandowski 0901.0933; Agullo, Ashtekar, Nelson 1302.0254; Gielen, Oriti 1709.01095; Gerhart, Oriti, Wilson-Ewing 1805.03099; ...








Challenges from the QG perspective:

- How to define (in)homogeneity?
- How to extract macroscopic dynamics?
- How to construct cosmological geometries?



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Relational description









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(T)GFT and Emergent Cosmology

Homogeneous cosmologies from (T)GFT condensates







Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma
angle = \mathcal{N}_{\sigma} \exp\left[\int \mathrm{d}^{d_{l}}\chi \int \mathrm{d}g_{a}\,\sigma(g_{a},\chi^{lpha})\hat{\varphi}^{\dagger}(g_{a},\chi^{lpha})
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angle$$

Collective states

LM, Oriti 2008.02774; LM, Oriti 2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238; Gielen 1404.2944;

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- Assuming $\sigma(g_a, \cdot) = \sigma(hg_ah', \cdot), \mathcal{D} = GL(3)/O(3) \times \mathbb{R}^{d_l}$:
- \mathcal{D} = space of spatial geometries + matter at a point.
- If χ^{μ} , $\mu = 0, \ldots, d-1$ constitute a matter ref. frame:

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Condensate Peaked States

• If σ is peaked on $\chi^{\mu} \simeq x^{\mu}$, $|\sigma\rangle_{\star}$ encodes relational info. about spatial geometry + matter at x^{μ} .

 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$

Relational strategy implemented at an effective level on "hydrodynamic" (averaged) quantities. ►

LM, Oriti 2008.02774; LM, Oriti 2112.12677; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Oriti, Sindoni 1311.1238; Gielen 1404.2944;

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Mean-field approximation: A non-linear and non-local extension of QC

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^{\dagger}]}{\delta \hat{\varphi}(g_a, x^0)} \right\rangle_{\sigma_{x^0}} = \int \mathrm{d} h_a \int \mathrm{d} \chi \, \mathcal{K}(g_a, h_a, (x^0 - \chi)^2) \sigma_{x^0}(h_a, \chi) + \lambda \frac{\delta V[\varphi, \varphi^*]}{\delta \varphi^*(g_a, x^0)} \bigg|_{\varphi = \sigma_{x^0}} = 0 \,.$$

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LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Pithis 2112.00091; ...

σ-hydrodynan

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LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Pithis 2112.00091; ...

Simplified σ -dynamics

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(T)GFT and Emergent Cosmology



 $\begin{array}{l} \mbox{Spatial relational homogeneity:}\\ \sigma \mbox{ depends on a single "clock" scalar field } \chi^0\\ (\mathcal{D} = \mbox{minisuperspace} + \mbox{clock}) \end{array}$

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Collective Observables

Number, volume (determined e.g. by the mapping with LQG) and matter operators (notation: $(\cdot, \cdot) = \int d\chi^0 dg_a$):

$\hat{N}=(\hat{arphi}^{\dagger},\hat{arphi})$	$\hat{m{V}}=(\hat{arphi}^{\dagger},m{V}[\hat{arphi}])$
$\hat{X}^{0}=\left(\hat{arphi}^{\dagger},\chi^{0}\hat{arphi} ight)$	$\hat{\Pi}^0 = -i(\hat{arphi}^\dagger,\partial_0\hat{arphi})$

• Observables \leftrightarrow collective operators on Fock space.

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(T)GFT and Emergent Cosmology

Relationality

• Averaged evolution wrt x^0 is physical:

Intensive
$$\langle \hat{\chi} \rangle_{\sigma_{\chi^0}} \equiv \langle \hat{\chi} \rangle_{\sigma_{\chi^0}} / \langle \hat{N} \rangle_{\sigma_{\chi^0}} \simeq x^0$$

- Emergent effective relational description:
 - Small clock quantum fluctuations.
 - Effective Hamiltonian H<sub>σ_{x⁰} ≃ ⟨Π̂⁰⟩_{σ_{x⁰}}.
 </sub>

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Wavefunction isotropy
$$\begin{split} \langle \hat{V} \rangle_{\sigma_X^0} &= \sum_{\upsilon} V_{\upsilon} |\tilde{\sigma}_{\upsilon}|^2 (x^0) \\ \langle \hat{N} \rangle_{\sigma_X^0} &= \sum_{\upsilon} |\tilde{\sigma}_{\upsilon}|^2 (x^0) \end{split}$$

Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Pithis 2112.00091;

Luca Marchetti

(T)GFT and Emergent Cosmology





Mean-field approximation

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Classical limit (large ρ_v s, late times)

• If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

 Quantum fluctuations on clock and geometric variables are under control.



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Bounce

- A non-zero volume bounce happens for a large range of initial conditions (at least one Q_v ≠ 0 or one E_v < 0).</p>
- The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

LM, Oriti 2008.02774; LM, Oriti 2010.09700; Oriti, Sindoni, Wilson-Ewing 1602.05881; Jercher, Pithis 2112.00091; ...

Luca Marchetti

(T)GFT and Emergent Cosmology

Exploring the physics of (T)GFT condensates

Physics of (T)GFT cosmology

Geometric acceleration from interactions

De Cesare, Oriti, Pithis 1606.00352; LM, Oriti 2112.12677; Oriti, Pang 2105.03751; Ladstätter, LM, Oriti (to appear); Jercher, LM, Pithis (to appear)

Physics of (T)GFT cosmology

Geometric acceleration from interactions

Early times: geometric inflation

Geometric inflation from QG interactions.

For some models bottom-up natural and slow-roll.

De Cesare, Oriti, Pithis 1606.00352; LM, Oriti 2112.12677; Oriti, Pang 2105.03751; Ladstätter, LM, Oriti (to appear); Jercher, LM, Pithis (to appear)
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Late times: phantom dark energy

 Phantom dark energy generated by QG effects (no field theoretic issue).

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Including more realistic matter: running couplings

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Including more realistic matter: running couplings

 Matching with GR requires the macroscopic constants (including G) to run with time.

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Connection with asymptotic safety?



- Classical system: gravity + 5 m.c.m.f. scalar fields. 4 of which constitute the relational frame.
- Perturbations at the level of σ.
- Matching with GR at late times only for super-horizon modes.



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Why the mismatch?

Perturbations at the level of σ.

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- Full relational frame requires quanta with different causal properties.
- Including quantum correlations substantially helps the matching.

De Cesare, Oriti, Pithis 1606.00352; LM, Oriti 2112.12677; Oriti, Pang 2105.03751; Ladstätter, LM, Oriti (to appear); Jercher, LM, Pithis (to appear)



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Geometry from quantum correlations!

De Cesare, Oriti, Pithis 1606.00352; LM, Oriti 2112.12677; Oriti, Pang 2105.03751; Ladstätter, LM, Oriti (to appear); Jercher, LM, Pithis (to appear)













Homogeneous, free

- Late times: FRLW flat classical dynamics.
- Early times: averaged quantum bounce.





Backup



Quite well understood from a classical perspective, less from a quantum perspective.

Isham 9210011; Rovelli Class. Quantum Grav. 8 297; Dittrich 0507106; Hoehn et al. 1912.00033 and 2007.00580; Tambornino 1109.0740; ...



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Notion of relationality can be classically encoded in relational observables:

- Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- Evolution in \(\tau\) is relational.
- *F_{f,T}(τ)* is a very complicated function, often written in series form.
- Applications only for (almost) deparametrizable systems, such as GR plus pressureless dust or massless scalar fields.

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Dirac approach: first quantize, then implement relationality

- Perspective neutral approach: all variables are treated on the same footing.
- Poor control of the physical Hilbert space.

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Reduced phase space approach: first implment relationality, then quantize

- No quantum constraint to solve.
- Led to quantization of simple deparametrizable models (LQG).
- Not perspective neutral. Too complicated to implement in most of the cases.

Isham 9210011; Rovelli Class. Quantum Grav. 8 297; Dittrich 0507106; Hoehn et al. 1912.00033 and 2007.00580; Tambornino 1109.0740; ...

Relational strategy and emergent quantum gravity theories



- ▶ Well understood from a classical perspective, less from a quantum perspective.
- Difficulties especially relevant for emergent QG theories.

LM. Oriti 2008.02774: Giulini 0603087: Kuchar Int.J.Mod.Phys.D 20(2011): Isham 9210011: Rovelli Class. Quantum Grav. 8 297:

Relational strategy and emergent quantum gravity theories



- Fundamental d.o.f. are weakly related to spacetime quantities;
- Set of collective observables;
- The latter expected to emerge from the former when a continuum limit is taken.
- Coarse grained states or probability distributions.

The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

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Effective approaches:

- Bypass most conceptual and technical difficulties;
- Relevant for observative purposes.

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Luca Marchetti

Emergent effective relational dynamics



LM, Oriti 2008.02774; Bojowald, Hoehn, Tsobanjan 1011.3040; Bojowald, Tsobanjan 0906.1772;

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Emergent effective relational dynamics



Concrete example: scalar field clock

Emergence

- Identify a class of states |Ψ⟩ which encode collective behavior and admit a continuum proto-geometric interpretation.
- Identify a set of collective observables:



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Emergent effective relational dynamics



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Effectivness

• It exists a "Hamiltonian" \hat{H} such that

$$rac{\mathrm{d}}{\mathrm{d}\langle\hat{\chi}\rangle_{\Psi}}\langle\hat{O}_{a}\rangle_{\Psi} = \langle [\hat{H},\hat{O}_{a}]\rangle_{\Psi} \ ,$$

and whose moments coincide with those of $\hat{\Pi}$.

$$\sigma_{\chi}^2 \ll 1$$
, $\sigma_{\chi}^2 \sim \langle \hat{N} \rangle_{\Psi}^{-1}$.

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