

Emergent Cosmology from (T)GFT condensates

based on 2008.02774, 2010.09700, 2110.11176 and 2112.12677; with D. Oriti, S. Gielen, A. Polaczek

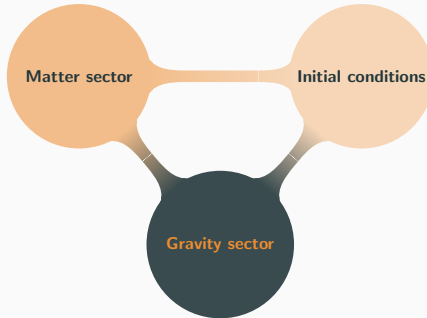
Luca Marchetti

QUAST seminar

OIST, 20 September 2022

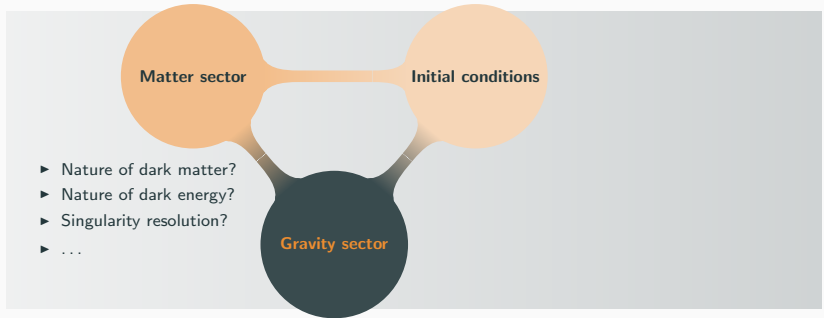
Arnold Sommerfeld Center
LMU Munich

The QG perspective on Cosmology

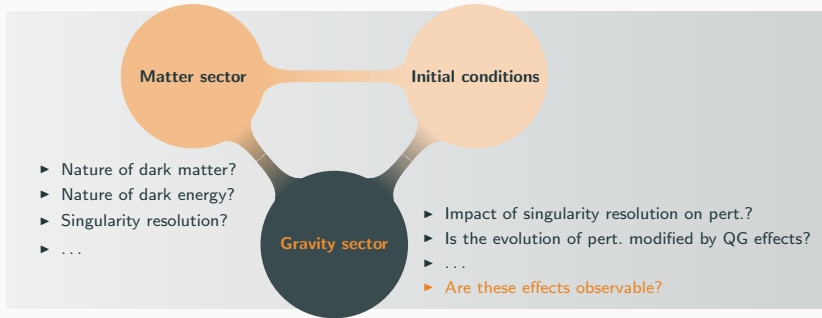


Ashtekar, Kaminski, Lewandowski 0901.0933; Agullo, Ashtekar, Nelson 1302.0254; Gielen, Oriti 1709.01095; Gerhart, Oriti, Wilson-Ewing 1805.03099; ...

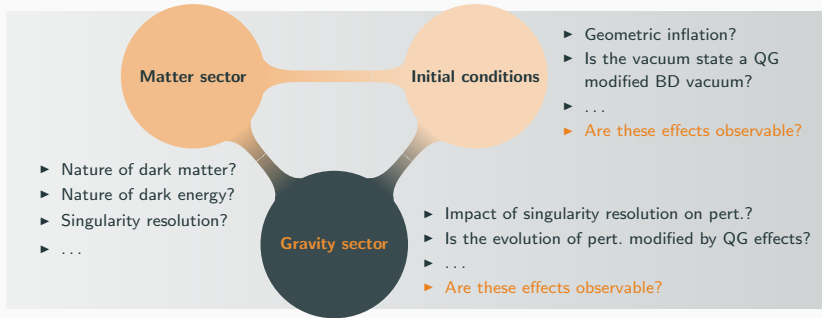
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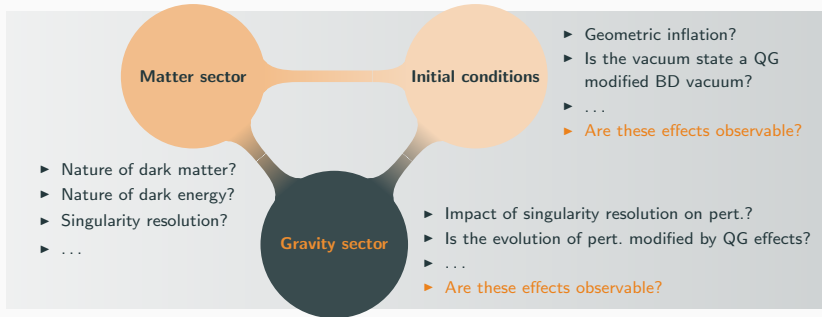
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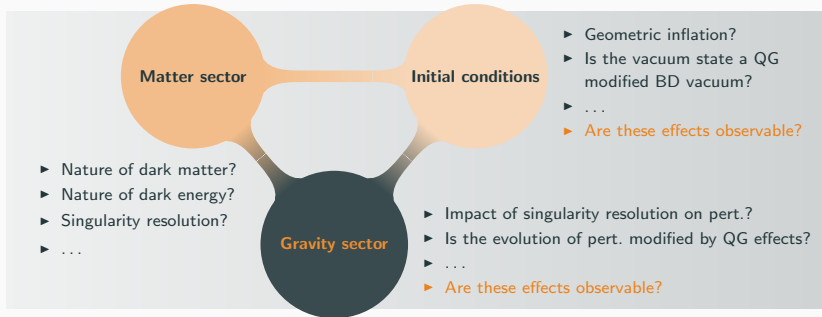
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Challenges from the QG perspective:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
- ▶ ...

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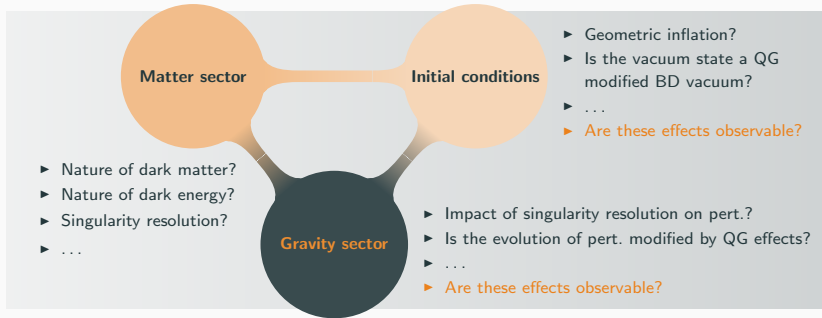


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Relational description

The QG perspective on Cosmology



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Relational description

Coarse-graining/
collective behavior

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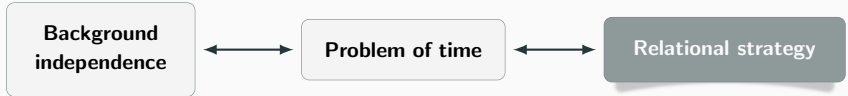
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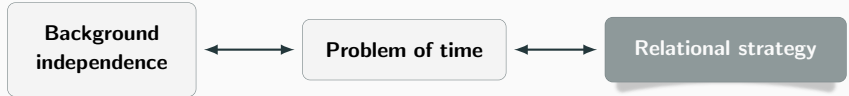
Relational dynamics

Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

Relational strategy: the classical and quantum GR perspective



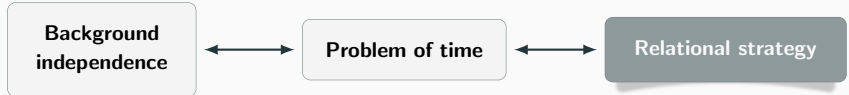
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Classical

Notion of relationality can be classically encoded in **relational observables**:

- ▶ Take two phase space functions, f and T with $\{T, C_H\} \neq 0$ (T relational clock).
- ▶ The relational extension $F_{f,T}(\tau)$ of f encodes the value of f when T reads τ .
- ▶ Evolution in τ is relational.
- ▶ $F_{f,T}(\tau)$ is a very complicated function, often written in series form.
- ▶ Applications only for (almost) deparametrizable systems, such as GR plus pressureless dust or massless scalar fields.

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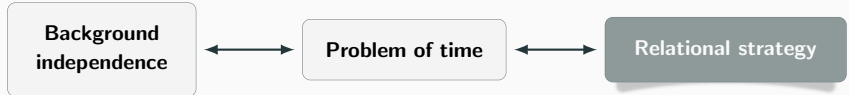
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Quantum GR

Dirac approach: first quantize, then implement relationality

- ▶ Perspective neutral approach: all variables are treated on the same footing.
- ▶ Poor control of the physical Hilbert space.

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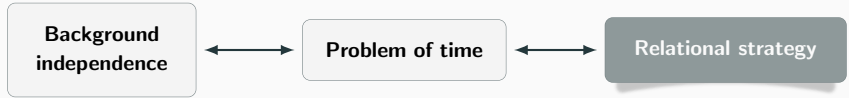
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Reduced phase space approach: first implment relationality, then quantize

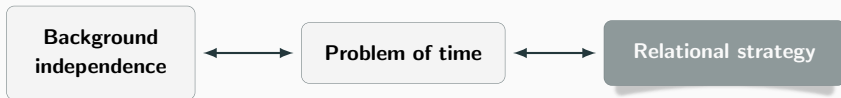
- ▶ No quantum constraint to solve.
- ▶ Led to quantization of simple deparametrizable models (LQG).
- ▶ Not perspective neutral. Too complicated to implement in most of the cases.

Relational strategy and emergent quantum gravity theories

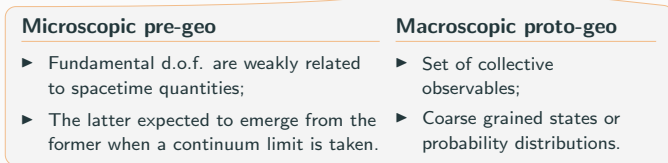


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- ▶ Difficulties especially relevant for **emergent** QG theories.

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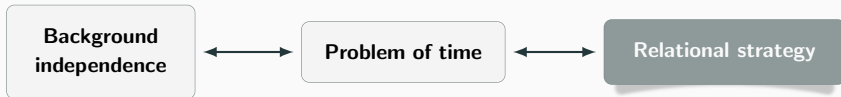


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The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

Relational strategy and emergent quantum gravity theories



- ▶ Well understood from a classical perspective, less from a quantum perspective.
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Microscopic pre-geo

- ▶ Fundamental d.o.f. are weakly related to spacetime quantities;
- ▶ The latter expected to emerge from the former when a continuum limit is taken.

Macroscopic proto-geo

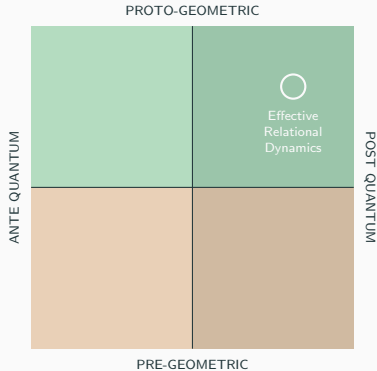
- ▶ Set of collective observables;
- ▶ Coarse grained states or probability distributions.

The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

Effective approaches:

- ▶ Bypass most conceptual and technical difficulties;
- ▶ Relevant for observative purposes.

Emergent effective relational dynamics

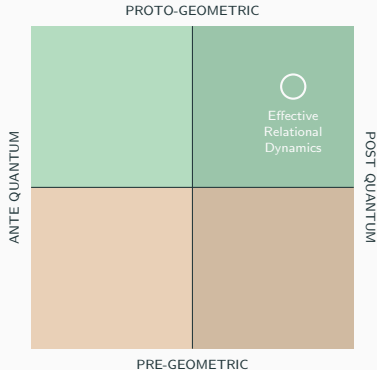


Basic principles

Emergence Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

Effectiveness Rel. evolution intended to hold on average. Internal clock not too quantum.

Emergent effective relational dynamics



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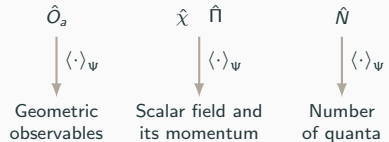
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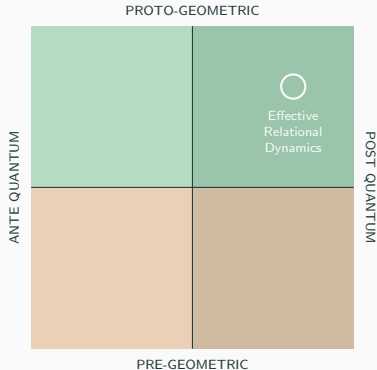
Concrete example: scalar field clock

Emergence

- Identify a class of states $|\Psi\rangle$ which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- Identify a set of collective observables:



Emergent effective relational dynamics



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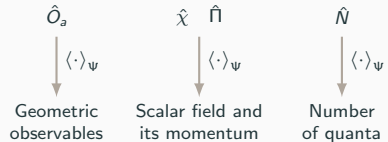
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Effectiveness

- It exists a “Hamiltonian” \hat{H} such that

$$i \frac{d}{d \langle \hat{\chi} \rangle_\Psi} \langle \hat{O}_a \rangle_\Psi = \langle [\hat{H}, \hat{O}_a] \rangle_\Psi ,$$

and whose moments coincide with those of $\hat{\Pi}$.

- Relative variance of $\hat{\chi}$ on $|\Psi\rangle$ should be $\ll 1$ and have the characteristic $\langle \hat{N} \rangle_\Psi^{-1}$ behavior:

$$\sigma_\chi^2 \ll 1, \quad \sigma_\chi^2 \sim \langle \hat{N} \rangle_\Psi^{-1} .$$

Homogeneous cosmologies from (T)GFT condensates

The (T)GFT approach to QG

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \rightarrow \mathbb{C}$ defined
on d copies of a group manifold G .

d is the dimension of the “spacetime to be” ($d = 4$)
and G is the local gauge group of gravity,
 $G = \text{SL}(2, \mathbb{C})$ or, in many applications, $G = \text{SU}(2)$.

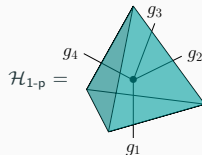
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Boundary states are $d - 1$ -simplices decorated with quantum geometric data:



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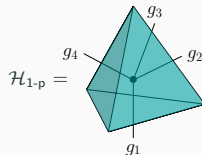
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- Group (Lie algebra) variables associated to discretized gravitational quantities.



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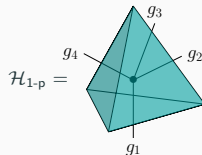
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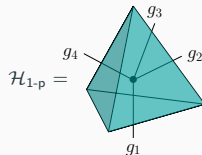
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S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

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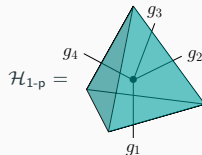
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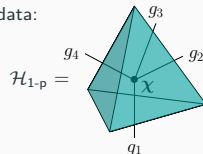
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Kinematics

Boundary states are $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.
- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Scalar field discretized on each d -simplex: each $d - 1$ -simplex composing it carries values $\chi \in \mathbb{R}^{d_I}$.



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S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + matter path integral.

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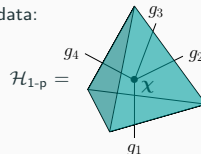
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GFTs are QFTs of atoms of spacetime.

QG condensates and peaked states

Extracting continuum physics

- Identify **coarse-grained states** and **collective observables** with a continuum interpretation.
- Obtain macroscopic, effective, and **relational** dynamics from the microscopic one.

Collective states

(T)GFT condensates

Simplest collective behavior: macroscopic σ dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[\int d^d \chi \int dg_I \sigma(g_I, \chi^\mu) \hat{\varphi}^\dagger(g_I, \chi^\mu) \right] |0\rangle$$

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- Assuming $\sigma(g_I, \cdot) = \sigma(hg_I h', \cdot)$, $\mathcal{D} = \text{GL}(3)/\text{O}(3) \times \mathbb{R}^d$:
- If χ^μ constitute a matter ref. frame: $\longrightarrow \sigma(g_I, \chi^\mu) \sim \text{distribution of spatial geometries at } \chi^\mu.$

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Relationality

(clac)Condensate Peaked States

- If σ is peaked on $\chi^\mu \simeq x^\mu$, $|\sigma\rangle_x$ encodes relational information about the spatial geometry at x^μ .
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- Peaking function e.g. Gaussian with non-zero width; reduced wavefunction assumed not to spoil peaking properties.

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0

Cosmology from QG condensates: observables and relationality

Spatial relational homogeneity:

σ depends on a single “clock” scalar field χ^0

Observables

Number, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation: $(\cdot, \cdot) = \int d\chi^0 d\mathbf{g}_I$):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

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- Averaged evolution wrt x^0 is physical:

$$\langle \hat{X} \rangle_\sigma \equiv \langle \hat{X} \rangle_\sigma / \langle \hat{N} \rangle_\sigma \simeq x^0$$

- ... and satisfies the requirements of effective relational dynamics (in the emergent limit).

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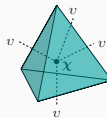
$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

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- $\langle \hat{O} \rangle_\sigma = O[\tilde{\sigma}]|_{\chi^0=x^0}$: exp. values of extensive operators are functionals of $\tilde{\sigma}$ localized at x^0 .

Wavefunction
 isotropy



Relationality

- Averaged evolution wrt x^0 is physical:

$$\langle \hat{X} \rangle_\sigma \equiv \langle \hat{X} \rangle_\sigma / \langle \hat{N} \rangle_\sigma \simeq x^0$$

- ... and satisfies the requirements of effective relational dynamics (in the emergent limit).

$$\langle \hat{V} \rangle_\sigma = \sum_v^f V_v |\tilde{\sigma}_v|^2(x^0) \quad v \text{ rep. label}$$

- $v = j \in \mathbb{N}/2$ for SU(2) (EPRL-like);
- $v = \rho \in \mathbb{R}$ for SL(2, \mathbb{C}) (ext. BC).

Mean-field approximation

- ▶ Mesoscopic regime: large N but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel.
- ▶ Isotropy: $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$ fundamental variables.

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(\mathbf{g}l, \mathbf{x}^0, \cdot)} \right\rangle_{\sigma_{\mathbf{x}^0}} = 0.$$

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Effective relational Friedmann equations

$$\left(\frac{V'}{3V} \right)^2 \simeq \left(\frac{2 \mathfrak{F}_v V_v \rho_v \text{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2 / \rho_v^2 + \mu_v^2 \rho_v^2}}{3 \mathfrak{F}_v V_v \rho_v^2} \right)^2 \quad \frac{V''}{V} \simeq \frac{2 \mathfrak{F}_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\mathfrak{F}_v V_v \rho_v^2}$$

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Classical limit (large ρ_v s, late times)

- ▶ If μ_v^2 is mildly dependent on v (or one v is dominating) and equal to $3\pi G$

$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

- ▶ **Quantum fluctuations** on clock and geometric variables are **under control**.

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Bounce

- ▶ A **non-zero volume bounce** happens for a large range of initial conditions (at least one $Q_v \neq 0$ or one $\mathcal{E}_v < 0$).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

Towards inhomogeneities

Scalar perturbations from (T)GFT condensates

Simplest (slightly) relationally inhomogeneous system

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Simplest (slightly) relationally inhomogeneous system

Classical

- ▶ 4 MCM **reference** fields (χ^0, χ^i) , with Lorentz/Euclidean invariant S_χ in field space.
- ▶ 1 MCM **matter** field ϕ dominating the e.m. budget and **relationally inhomog.** wrt. χ^i .

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Quantum

- ▶ (T)GFT field: $\varphi(g_I, \chi^\mu, \phi)$, depends on 5 discretized scalar variables.
- ▶ EPRL-like or extended BC model with S_{GFT} respecting the classical matter symmetries.

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Only isotropic info: $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

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States

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- Averaged q.e.o.m. \longrightarrow coupled differential equations for ρ and θ .
- Decoupling for a range of values of CPSs and large N (late times).



Dynamic equations
for $\langle \hat{V} \rangle_\sigma, \langle \hat{\Phi} \rangle_\sigma$

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- Super-horizon GR matching.
- **No matching** for intermediate modes (because of different coupling with bkg effective metric)!
- Effective metric signature determined by CPSs.

A state-agnostic approach

Effective approach for constrained quantum systems

How does our scheme for extraction
of relational cosmological physics
depend on the specific choice of states?



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Step 1: definition of the quantum phase space

- Describe the system with exp. values $\langle \hat{A}_i \rangle$ and moments:
- Poisson structure inherited from the algebra structure

$$\left\{ \langle \hat{A}_i \rangle, \langle \hat{A}_j \rangle \right\} = (i\hbar)^{-1} \left\langle [\hat{A}_i, \hat{A}_j] \right\rangle \quad (\text{same for } \Delta s).$$

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Step 3: relational rewriting

- Determine the remaining gauge flow which preserves the gauge conditions.
- Write evolution of the remaining variables wrt. T (classical clock).

A state agnostic approach: application to (T)GFT

How can this framework be generalized to a **field theory context**?

Infinitely many algebra generators.

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Motivations

- ▶ Interest in a coarse grained system characterized by a small number of macroscopic (1-body) observables.
- ▶ Expected to be the case for cosmology.

Coarse-graining truncation

- ▶ When the e.o.m. are linear, consider an integrated 1-body quantum constraint.
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- ▶ Quantum constr. $\hat{C} = \int \hat{\varphi}^\dagger \mathcal{D}\hat{\varphi} = m^2 \hat{N} - \hat{\Lambda} - \lambda \hat{\Pi}_2$
- ▶ Generators: $\hat{X}, \hat{\Pi}, \hat{\Pi}_2, \hat{N}, \hat{\Lambda}$ and \hat{K} .
- ▶ \hat{K} such that $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$.

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- ▶ \hat{K} such that $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$.
- ▶ The procedure can naturally be carried over by choosing as clock variable \hat{K} .
- ▶ Relational evolution of $\langle \hat{X} \rangle$ in agreement with classical cosmology.
- ▶ Fluctuations are decoupled from expect. values.
- ▶ If they are small at small $\langle \hat{K} \rangle$ they stay small even at large $\langle \hat{K} \rangle$ (probably associated to the constancy of \hat{N}).

Results

Conclusions

Results and perspectives

Relational Dynamics and Emergent QG

Conclusions

- ✓ Presentation of a scheme to define effective relational dynamics for emergent QG theories.
- ✓ Concrete realization in (T)GFT cosmology.
- ✓ The interplay between quantum effects, emergence and relationality was highlighted.

Perspectives

- ▶ How to change frame in an effective relational context? (Crucial to study unitarity and frame covariance.)
- ▶ When are QG material frames ideal RFs?
- ▶ Extension of state-agnostic approach to QFT?

Cosmology from Full QG





Conclusions

- ✓ **Bkg**: Effective volume dynamics with correct classical limit and possible singularity resolution.
- ✓ **Bkg**: Investigation of the impact of quantum fluctuations.
- ✓ **Bkg**: (almost) state-agnostic extraction of cosmological relational dynamics.
- ✓ **Pert**: First steps towards a relational cosmological perturbation theory **from full QG**:

✓ Super-horizon matching with GR.

✗ No matching with GR at intermediate scales.

Perspectives

- ▶ **Bkg**: Inclusion of different matter fields, e.g. scalar field with potential. 
- ▶ **Pert**: Impact of bounce on perturbations. 
- ▶ **Pert**: Investigate sub-horizon GR mismatch. 
 - What gravity models do match?
 - Model building? Breakdown of approximations?
- ▶ **Pert**: Study out of condensate perturbations. 
- ▶ **Pert**: Reconstruct an effective metric (produce operators with geometric macro-interpretation).

Backup

Volume at late times

Background

Classical

- ▶ Harmonic gauge: $N = a^3$.
- ▶ Negligible contribution of reference matter.

$$(\bar{V}'/\bar{V})^2 = 12\pi G \pi_\phi^{(c)}$$

$$(\bar{V}'/\bar{V})' = 0$$

Quantum

- ▶ Wavefunction peaked on $\pi_\phi = \tilde{\pi}_\phi$.
- ▶ Domination of single spin v_o .
- ▶ $\mu_{v_o}(\pi_\phi) \simeq c_{v_o} \pi_\phi$, with $4c_{v_o}^2 = 12\pi G$.

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Perturbations

Classical

- ▶ First order harmonic gauge.
- ▶ Negligible contribution of reference matter.
- ▶ Define $V(x) = \sqrt{\det q_{ij}} \equiv \bar{V} + \delta V$.

$$\delta V'' - 6\mathcal{H}\delta V' + 9\mathcal{H}^2\delta V - \bar{V}^{4/3}\nabla^2\delta V = 0.$$

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- ▶ Wavefunction peaked on $\pi_\phi = \tilde{\pi}_\phi$.
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Super-horizon

- ▶ Matches the classical solution $\delta V \propto \bar{V}$.

Sub-horizon

- ▶ Same diff. structure but different powers of \bar{V} .

No matching with GR for arbitrary modes.

Matter at late times

Background

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- ▶ Harmonic gauge: $N = a^3$.
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$$\pi_{\phi}^{(c)} = \text{const.}.$$

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$$\langle \hat{N}_{\phi} \rangle_{\bar{\sigma}} = \tilde{\pi}_{\phi} \bar{N},$$

$$\langle \Phi \rangle_{\bar{\sigma}} = \left[-\partial_{\pi_{\phi}} \left[\frac{Q_{v_o}}{\mu_{v_o}} \right] + Q_{v_o} \frac{\partial_{\pi_{\phi}} \mu_{v_o}}{\mu_{v_o}} x^0 \right]_{\pi_{\phi} = \tilde{\pi}_{\phi}}$$

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Matching conditions

- ▶ $\pi_{\phi}^{(c)} \equiv \langle \hat{\Pi}_{\phi} \rangle_{\bar{\sigma}} / \bar{N} = \tilde{\pi}_{\phi}$.
- ▶ $\phi \equiv \langle \hat{\Phi} \rangle_{\bar{\sigma}} = -c_{v_o}^{-1} + \tilde{\pi}_{\phi} x^0$, $Q_{v_o} \simeq \pi_{\phi}^2$!
- ▶ Peaking in $\pi_{\phi} \rightarrow$ peaking in matter field momenta.
- ▶ Emergent G related to matter content!

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- ▶ Domination of single spin v_o : $\delta V \equiv 2\bar{\rho}_{v_o} \delta\rho_{v_o}$.

$$\delta\phi = \delta V / \bar{V} + \bar{N} [\partial_{\pi_{\phi}} \theta_{v_o}]_{\pi_{\phi} = \tilde{\pi}_{\phi}}.$$

- ▶ Matching at super-horizon scales
- ▶ No matching for intermediate scales.