Landau-Ginzburg analysis of (T)GFT models

(based on 2112.12677 and 2209.04297; in collaboration with D. Oriti, A. Pithis, J. Thürigen)

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Arnold Sommerfeld Center
LMU Munich

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Introduction to (T)GFT

The (T)GFT approach to quantum gravity

(Tensorial) Group Field Theories:
theories of a field $\varphi : G^d \to \mathbb{C}$ defined on d copies of a group manifold G.

d is the dimension of the "spacetime to be" ($d = 4$) and G is the local gauge group of gravity, $G = \mathrm{SL}(2, \mathbb{C})$ or, in many applications, $G = \mathrm{SU}(2)$.

Oriti 0912.2441; Oriti 1408.7112; Krajewski 1210.6257; Gielen, Oriti 1311.1238; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Sindoni 1602.08104; ...

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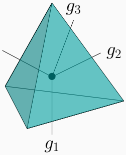
Kinematics

Boundary states are $d - 1$-simplices decorated with quantum geometric data:

$\mathcal{H}_{\text{1-p}} =$ 

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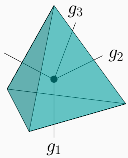
$\mathcal{H}_{\text{1-p}} =$ 

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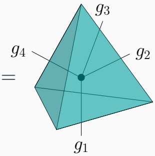

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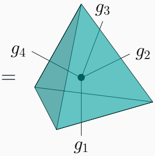
- Group (Lie algebra) variables associated to discretized gravitational quantities. $\mathcal{H}_{\text{1-p}} =$ 

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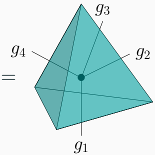

Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity path integral.

$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

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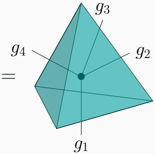

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- Γ are dual to spacetime triangulations.

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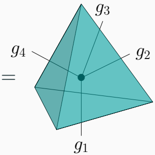

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GFTs are QFTs of atoms of spacetime.

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Li, Oriti, Zhang 1701.08719; Oriti 0912.2441; Gielen, Oriti 1311.1238; Oriti, Sindoni, Wilson-Ewing 1602.05881; Gielen, Sindoni 1602.08104; ...

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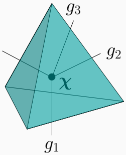
- Geometricity constraints imposed analogously as before.
- Scalar field discretized on each d-simplex: each $d-1$-simplex composing it carries values $\chi \in \mathbb{R}^{d_l}$.

$\mathcal{H}_{1\text{-p}} =$



(T)GFT and matter: scalar fields

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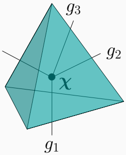
$\mathcal{H}_{\text{1-p}} =$ 

Dynamics

S_{GFT} obtained by comparing Z_{GFT} with simplicial gravity + scalar fields path integral.

- Geometric data enter the action in a non-local and combinatorial fashion.
- Scalar field data are local in interactions.
- For minimally coupled, free, massless scalars:

$$\mathcal{K}(g_I, g_J; \chi, \chi') = \mathcal{K}(g_I, g_J; |\chi - \chi'|^2)$$
$$\mathcal{U}(g_I^{(1)}, \dots, g_I^{(5)}, \chi) = \mathcal{U}(g_I^{(1)}, \dots, g_I^{(5)})$$

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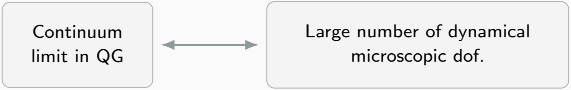
Continuum physics and QG: the general perspective

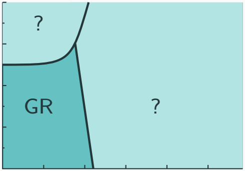


Oriti 2112.02585, Reuter, Saueressig 2019, Kopietz et al. 2010, Finocchiaro, Oriti 2004.07361, Carrozza 1603.01902 . . .

Continuum physics and QG: the general perspective

Continuum limit in QG ↔ Large number of dynamical microscopic dof.

Full partition function needed!



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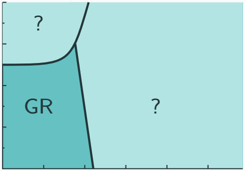

The (F)RG perspective

Spacetime QFT	(T)GFT

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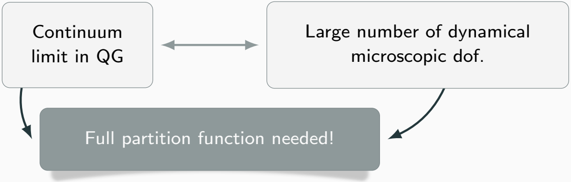

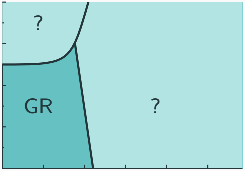

The (F)RG perspective

Spacetime QFT

- Energy scale defines the flow from IR and UV.

(T)GFT

- Only group structures as scales (e.g. rep. labels).

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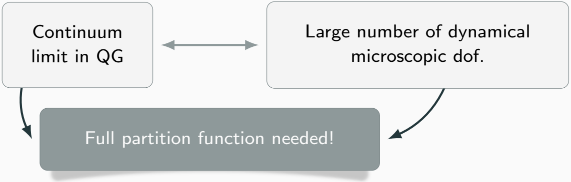

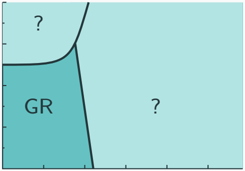

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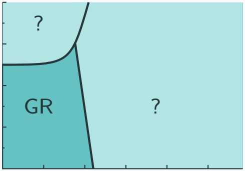

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▶ Theory space constrained by symmetries.	▶ Symmetries of (T)GFT models not well known.

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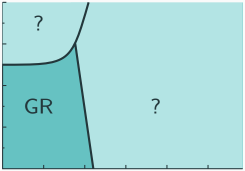

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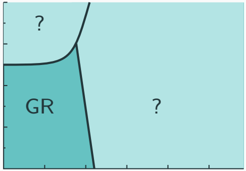

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Landau-Ginzburg theory

- Based on collective quantity: order parameter.
- Mean-field (saddle-point) approx. of Z: "simple" computations!
- Good description of quantum phase transitions.
- Allows to study critical behavior of Gaussian fluctuations over homogenenous mean-field.

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Landau-Ginzburg theory for toy models

Landau-Ginzburg for (T)GFT toy models with matter

Can a condensate phase actually be realized?

What is the impact of quantum fluctuations?

Answers within the Landau theory for abelian models

LM, Oriti, Pithis, Thürigen 2110.15336 ; Pithis, Thürigen 2007.08982; Pithis, Thürigen 1808.09765;

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Quantum fluctuations

Mean field theory is reliable for d_G such that

$$2V/(V-2) \leq d_I + d_G(d - s_0)$$

- Ginzburg criterion $\left\langle (\delta\Phi)^2 \right\rangle_\Omega \ll \left\langle \Phi_0^2 \right\rangle_\Omega$

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Matter matters!

Matter d.o.f. drastically affect the critical behavior of the system:

- Large correlations associated to the symmetry breaking point when matter is included!
- Matter lowers critical dimension and improves mean field theory!

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Analysis of a condensate transition characterized by breaking of global $\mathbb{Z}_2$-symmetry.

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Analysis of a condensate transition characterized by breaking of global $\mathbb{Z}_2$-symmetry.

- Φ order parameter, S effective action, d rank.
- d_G group dimension, Δ Laplacian on (abelian) group.
- μ bare mass: $\mu \to 0$ signals the transition.

$$S[\Phi] = (\Phi, \mathcal{K}\Phi) + \sum_{\gamma} \lambda_\gamma \int_{\mathbb{R}^{d_l}} \mathrm{d}\boldsymbol{\chi} \; \mathrm{tr}_\gamma(\Phi) \,,$$

$$\mathcal{K} = -\sum_{i=1}^{d_l} \alpha_i \partial^2_{\chi_i} + \sum_{c=1}^{d} (-1)^{d_G} \Delta_c + \mu \,,$$

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- s_0 lowest number of zero modes; same FRG scaling.

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Landau-Ginzburg theory for geometric models

Landau-Ginzburg for (T)GFT geometric models with matter

Can a condensate phase actually be realized?

What is the impact of quantum fluctuations?

Answers within the Landau theory for geometric models

Procedure identical as before, except for a few technical complications

- More complicated group theoretic/representation structure.
- Less straightforward regularization scheme needed.
- Imposition of closure and simplicity constraints.

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- Infinite scaling dimension!

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Matter matters, but only at low curvature

Due to hyperbolic properties of SL$(2, \mathbb{C})$ matter effects are in general washed out.

- If the hyperbolic space is "flattened", the exponential suppression of Q becomes polynomial.
- In this case, matter lowers critical dimension and improves mean field theory, as before.

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Setting

- IR regulator Λ and curvature scale a (usually $a = 1$).

Three important regimes

$\Lambda = 2\pi a$

- Reduces to Spin(4) after Wick rotation $A^+_\Lambda \to T^+$.
- Intermediate comp. done in this regime.

$\Lambda \to \infty$

- Limit and back Wick rotation at the end.

a finite

- $\mathrm{SL}(2,\mathbb{C})$.

Λ/a finite

- Flat case.

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- Mean-field analysis as before.
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- Computation of ξ possible: only asymptotic behavior of correlations necessary.

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Ginzburg criterion

$$Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{V_\gamma}{V_\gamma - 2} - \frac{d_l}{2}} e^{-2(4-s_0)\xi/a}$$

- In the IR (large ξ/a), infinite scaling dimension.
- In the flat limit $Q \sim \lambda_\gamma^{\frac{2}{V_\gamma - 2}} \xi^{\frac{2V_\gamma}{V_\gamma - 2} - d_l - 3(4-s_0)}$.

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Conclusions

Summary of results and a few perspectives

Results: Matter

- ✓ From a QG perspective, scalar field data enter locally in the (T)GFT action.
- ✓ Matter makes the (T)GFT domain non-compact, allowing phase transitions.
- ✓ Critical behavior wrt matter data same as for local field theories on spacetime.
- ✓ In general, matter improves validity of mean-field theory.

Results: Geometry

- ✓ Non-locality affects scaling properties via the lowest number of zero modes.
- ✓ Due to hyperbolicity, scaling dimension is infinite (mean-field theory always valid).
- ✓ Closure constraint does not affect critical behavior in general by $d_G \to d_G - 1$.
- ✓ Matter "trumped" by geometry, unless $a \to \infty$.

Perspectives

- Extend the analysis to non-uniform mean-field config. (cosmology).
- Study time- and light-like tetrahedra.
- Only IR Gaussian fixed point? FRG!
- Use matter data to define scales and locality in FRG?